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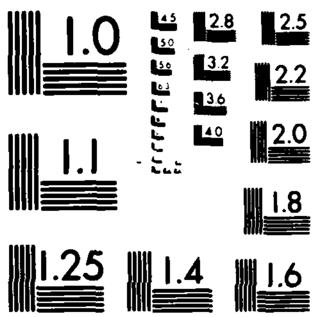
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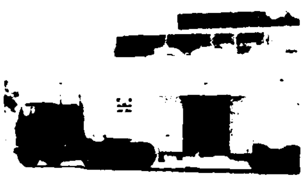

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DEVELOPMENT OF PROBABILISTIC RIGID PAVEMENT DESIGN METHODOLOGIES FOR MILITARY AIRFIELDS

by

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Final Report

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The current Corps of Engineers design procedure for rigid airfield pavements is based on the Westergaard free edge stress slab theory, and a proposed procedure is based on the multilayer elastic theory. These two design procedures have been expanded to airfield pavement designs expressed in probabilistic and reliability terms. Further developments were required in these procedures to make the analysis more practicable. Two major investigations were conducted:

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20. ABSTRACT (Continued).

(a) Evaluation and use of the composite modulus of elasticity for layers beneath the rigid pavement,

(b) Evaluation of the maximum tensile stress at the bottom of the slab for different aircraft types.

Derivations obtained from the investigation of the composite modulus and maximum tensile stress are reported and are included in computer programs for probabilistic/reliability analysis of rigid pavements. The approximate closed form (Taylor series expansion) is utilized. Example runs of the computer program are presented.

PREFACE

The work reported herein was performed for the U. S. Army Engineer Waterways Experiment Station (WES) under Purchase Order DACA39-82-M-0074 with the University of Maryland during the period May 1982 to June 1983. The work was funded by the Office, Chief of Engineers, U. S. Army, under the FY 82 RDTE Program, Project: 4A161102AT22, Task AO, Work Unit 009, "Methodology for Considering Material Variability in Pavement Design." OCE Project Monitor was Mr. S. S. Gillespie. The work was conducted and report prepared by Drs. M. W. Witczak and J. Uzan and Mr. M. Johnson.

Dr. W. R. Barker, Pavement Systems Division (PSD), Geotechnical Laboratory (GL), WES, was the Principal Investigator. The study was conducted under the general supervision of Dr. T. D. White, Chief, PSD, and Dr. W. F. Marcuson III, Chief, GL.

Commander and Director of WES during the period of this study was COL Tilford C. Creel, CE. Technical Director was Mr. F. R. Brown.

CONTENTS

VOLUME I

STATE OF THE ART VARIABILITY OF

AIRFIELD PAVEMENT MATERIALS

TABLE OF CONTENTS

LIST OF TABLES

Chapter 1

INTRODUCTION

At present, pavement design methodologies are deterministic, that is, a unique pavement system is designed for the set of input variables having unique values. The variations of these design parameters have been taken into account either by setting limiting values on the parameters or by using a factor of safety. Such an approach, however, leaves the engineer without any true quantitative estimate of overall design reliability. A probabilistic design method may therefore be developed and used in order to take material variabilities into account within a reliability framework. The use of a probabilistic methodology necessitates the determination of not only the mean (average) value of each parameter; but a measure of the variability of the specific parameter in question (i.e., standard deviation, variance, coefficient of variation).

The purpose of this report is to give a general summary of values or ranges of values, found from a literature search, that describe material variabilities. This report summarizes a state-of-the-art review in variabilities of material properties which are used in pavement design. This study specifically concentrated on normal design input variables commonly used in rigid pavement airfield studies. As such, the wealth of variability information concerning such non-design parameters (i.e., soil classification, gradation, component variability analyses, etc.) was not included.

It should be noted that, in some instances, the standard deviation is the given measure of variability. Since the coefficient of variation may not always be constant, the standard deviation may give a better

indication of the variability of the material. If a standard deviation is given and the coefficient of variation is required, it may be obtained by dividing the standard deviation by the mean ($CV = \frac{\sigma}{\bar{X}} \times 100\%$).

Chapter 2 contains brief discussions of material properties and their variation while Chapter 3 contains the summary and recommendations of values to be used within a probabilistic design method for rigid airfield pavements.

Chapter 2

MATERIAL VARIABILITY

Well defined variability information is more prevalent from highway pavements than for airfield pavements. However, there is little reason to believe that there are major differences in the variability of the two types of pavements. Variabilities, therefore, have been collected for both highways and airfields where there is insufficient airfield information.

PORTLAND CEMENT CONCRETE MATERIALS

Thickness

There is very little variation in the thickness of Portland Cement concrete pavements. Table 1 shows information taken from several highway and airfield pavements. The coefficients of variation are generally below 3% but go as high as 8%. Table 2 shows information from the Louisiana Department of Highways. Here, the coefficients of variation are all less than 5%. Information could only be found for two airfield pavement sections (Table 1). All of the information gathered was for pavement thicknesses of less than 15 inches. An insignificant amount of data was found for pavements of larger thicknesses and therefore no conclusions may be drawn.

Modulus of Elasticity

There was a sufficient amount of information available on the modulus of elasticity for airfields. A summary of the mean values and the

Table 1. Thickness of Portland Cement Concrete Pavements (Kennedy, Hudson and McCullough

	Project Identification	Sample Plan**		Number of Cores	Pavement Thickness* Mean (inches)	CV (%)	Design (inches)
Highways	2-A	ATP		38	8.3	2.6	8.0
	2-E	ATP		50	8.3	2.5	8.0
	17-B	ATP		50	8.2	2.6	8.0
		Cluster		10	7.8	1.2	8.0
	17-M	ATP		47	8.2	2.6	8.0
		Cluster 1		7	7.7	1.0	8.0
		Cluster 2		8	7.6	1.1	8.0
	18-N	ATP		9	8.8	3.7	8.0
	18-0	ATP		24	9.5	4.7	9.0
	19-A	ATP		34	8.2	2.9	8.0
	19-B	ATP		31	8.2	3.4	8.0
		Cluster 1		10	7.6	0.6	8.0
		Cluster 2		9	7.6	1.4	8.0
		Weighted Average excluding clusters			-	2.9	-
		Variation	Limits		-	2.5-4.7	-
			Range		-	2.2	-
Airports	O'Hare Rwy 9R-27L	ATP		12	10.9	8.3	10.0
	O'Hare Rwy 4R-22L	ATP		10	14.8	3.3	14.0
		Weighted Average			-	6.0	-
		Variation	Limits		-	3.3-8.3	-
			Range		-	5.0	-

* Thickness determined by measuring height of core in laboratory.

** Along-the-pavement or cluster samples from thin section where thickness is less than design value.

TABLE 2. Summary of Statistical Results on Thickness *of Concrete Pavement* (Kennedy, Hudson and McCullough)

Project Number	Number of Samples	Mean (inches)	Variance (inches)	Standard Deviation (inches)	CV (%)
		8-In. Uniform Thickness			
1	34	8.66	0.192	0.435	5.0
2	39	8.42	0.171	0.415	4.9
3	48	8.35	0.040	0.200	2.4
4	58	8.36	0.077	0.276	3.3
5	61	8.05	0.035	0.185	2.3
6	66	8.11	0.089	0.300	3.7
7	73	8.06	0.112	0.333	4.1
Pooled Values	--	8.29	0.088	0.300	3.6
		9-In. Uniform Thickness			
1	35	9.25	0.0466	0.210	2.3
2	51	9.19	0.121	0.350	3.8
3	58	9.28	0.048	0.220	2.4
4	65	9.18	0.060	0.240	2.6
5	74	9.20	0.185	0.430	4.7
6	88	9.11	0.029	0.170	1.9
Pooled Values	--	9.20	0.083	0.290	3.1
		10-In. Uniform Thickness			
1	64	10.38	0.061	0.240	2.3
2	124	10.34	0.079	0.280	2.7
3	132	10.35	0.079	0.230	2.2
4	141	10.28	0.083	0.270	2.8
Pooled Values	---	10.34	0.069	0.270	2.6

coefficients of variation may be found in Table 3. Mean values ranged from 2.90×10^6 psi to 3.89×10^6 psi with an average of 3.45×10^6 psi. Coefficients of variation ranged from 21 to 49 with an average of 34.4. Tables 4a, 4b, and 4c are the results of a series of laboratory tests. These results show that the modulus value reaches a slight peak at an age of 60 days and then decreases slightly with age. The mean modulus value was 4.54×10^6 psi with a range of 4.05×10^6 psi to 5.16×10^6 psi. The standard deviation increased with time until the 60 day mark was reached and then it decreased slightly. The standard deviation ranged from 0.06×10^6 psi to 0.22×10^6 psi. The coefficient of variation ranged from 1.2% to 4.8%.

Poisson's Ratio

Tables 4, 5, and 6 are a summary of data from Gibeaut (4), who investigated the Poisson's Ratio of PCC. Values tended to decrease with time whereas the standard deviation tended to increase. There was no true trend for the coefficient of variation. The mean value of Poisson's ratio ranged from 0.162 to 0.212 with an average of 0.188. The standard deviation ranged from 0.036 to 0.019 and the coefficient of variation ranged from 9.3% to 20.2%.

TABLE 3. Summary of Test Results for Cored Specimens from PCC Airfield Pavements (Kennedy)

Airport	Project Identification	Number of Tests	Indirect Modulus of Elasticity Mean (10^6 psi)	CV (%)
O'Hare	Runway 9R-27L	48 47	3.39*	34
		15	3.33*	28
	Runway 4R-22L	20	2.90**	26
		39	3.89*	49
Palmdale	Runway 7-25 (Existing Pavement)	11	-	-
	Runway 7-25 (Overlay)	38 39	3.86*	32
	Taxiway A Overlay	7 8	3.39*	40
	Taxiway B Overlay	20	3.09*	21
Midway	Runway 4R-22L 13R-31L	11	3.21**	43
Richmond	Taxiway 5-4 Runway 2	16	3.02**	31
	Weighted	Average	3.45	34.4
	Variation	Limits	2.90 - 3.89	21 - 49
		Range	0.99	28

* Assumed Poisson's ratio = 0.20

** Assumed Poisson's ratio = 0.15

Table 4

Individual Sonic E and μ Values

(Expanded from Gibeaut)

	50 Series						
Age Days	E x 10^6 psi	$\bar{E}$ x 10^6 psi	σ_E x10^6psi	μ	$\bar{\mu}$	σ_μ	CV %
14	4.71 4.88 4.86 5.07	4.88	0.15	0.176 0.201 0.217 0.215	0.202	0.019	9.41
28	4.72 4.83 4.95 5.10	4.90	0.16	0.147 0.175 0.201 0.211	0.184	0.029	15.76
60	4.74 4.84 5.00 5.16	4.94	0.18	0.153 0.173 0.218 0.229	0.193	0.036	18.65
90	4.68 4.83 4.89 5.14	4.89	0.19	0.140 0.162 0.186 0.224	0.178	0.036	20.22
159	4.71 4.82 4.76 5.08	4.84	0.16	0.143 0.172 0.164 0.208	0.172	0.027	15.75

Table 5

Individual Sonic E and μ Values
(Expanded from Gibeaut)

Age Days	60 Series						CV %
	$E \times 10^6$ psi	$\bar{E} \times 10^6$ psi	$\sigma_c \times 10^6$ psi	μ	$\bar{\mu}$	σ_μ	
14	4.64 4.40 4.69 4.29	4.51	0.91	0.191 0.186 0.223 0.170	0.193	0.022	11.40
28	4.72 4.40 4.81 4.40	4.58	0.21	0.181 0.139 0.204 0.170	0.174	0.027	15.52
60	4.72 4.46 4.80 4.41	4.60	0.19	0.168 0.133 0.190 0.169	0.165	0.024	14.55
90	4.70 4.40 4.77 4.36	4.56	0.21	0.170 0.124 0.193 0.162	0.162	0.029	17.90
152	4.76 4.45 4.76 4.34	4.58	0.22	0.187 0.135 0.184 0.174	0.170	0.024	14.12

Table 6

Individual Sonic E and μ Values
(Expanded from Bibeaut)

		70 Series					
Age Days	$E \times 10^6$ psi	$\overline{E} \times 10^6$ psi	$\sigma_c \times 10^6$ psi	μ	$\overline{\mu}$	σ_μ	CV %
14	4.07 4.14 4.18 4.17	4.14	0.05	0.263 0.203 0.187 0.193	0.212	0.035	16.51
28	4.06 4.18 4.20 4.19	4.16	0.07	0.235 0.190 0.187 0.212	0.206	0.022	10.68
60	4.05 4.24 4.19 4.20	4.17	0.08	0.240 0.193 0.187 0.226	0.212	0.026	12.26
90	4.05 4.24 4.15 4.17	4.15	0.08	0.215 0.191 0.172 0.212	0.198	0.020	10.10
130	4.05 4.20 4.10 4.15	4.13	0.06	0.218 0.191 0.161 0.211	0.195	0.025	12.82

Modulus of Rupture

Current rigid pavement design methods utilize the coefficient of variation in determining the design modulus of rupture. Values of C.V. used are:

< 10% for excellent construction control

10-15% for good control

15-20% for fair control

> 20% for poor control

Tables 7 and 8 show that in most cases excellent construction control may be maintained.

ASPHALT CONCRETE MATERIALS

Thickness

As for the thickness of PCC pavements, there is little information on the wide range of pavement thicknesses. Yoder and Witczak suggest a typical standard deviation range for new highway construction 0.3 to 0.8 inches for asphalt concrete pavements.

Dynamic Modulus

Table 9 is a summary of laboratory tests which were conducted to determine the dynamic modulus of asphalt concrete. It may be observed that the coefficient of variation within a project is a function of temperature and that there is no specific trend of frequency. It should also be noted that the in-situ (field) variability should be greater than the laboratory variability. In general, ranges of the CV values as a function of temperature are: (9%-16%) (11%-19%) and (20%-23%) for temperatures of 40°F, 70°F and 100°F, respectively.

Table 7

Average 7-Day Flexural Strength and Standard Deviations,

Portland Cement Concrete Pavement (Brown)

No. of Samples, n	Strength, psi	Standard Deviation $\hat{\sigma}$, psi	Coefficient of Variation (%)
414	688	73	10.6
74	552	50	9.1
324	713	77	10.8
76	642	75	11.7
8	679	30	4.4
42	645	18	2.8
44	705	64	9.1
170	736	55	7.5
38	680	120	17.6
16	591	52	8.8
41	676	92	13.6
8	640	55	8.6
1255	Avg. 662	63	9.5

Table 8

Average 28-Day Flexural Strength and Standard Deviations, Portland Cement Concrete Pavement (Brown)

No. of Samples, n	Avg Flexural Strength, psi	Standard Deviation σ, psi	of Variation(%)
582	781	61	7.8
146	719	57	5.1
312	862	83	9.6
101	753	70	9.3
82	774	34	4.4
26	734	26	3.5
735	739	66	8.9
67	828	122	4.7
16	688	40	5.8
82	840	68	8.1
8	717	60	8.4
2157	Avg 766	61	8.0

TABLE 9 Lab Variability - Asphalt Concrete

A.C. Dynamic Modulus Tests (E°)(from The Asphalt Institute)

Test Property	Test Conditions		Test Variability		Within Project		Remarks
	Temp.(°F)	Freq.(Hz)	s(X 10^3 psi)	C.V.	S(X 10^3 psi)	C.V.	
A.C. Dynamic Modulus E° (psi)	40	1	53.8	4.1	162.0	12.5	4 Field projects
		4	122.0	7.7	145.5	9.2	3 Cores per project
		16	64.0	3.5	294.0	16.0	2 Replicates per core
	70	1	26.4	6.2	71.5	16.7	
		4	17.8	2.9	117.5	19.0	
		16	28.3	3.6	88.8	11.3	
	100	1	5.8	4.7	27.4	22.3	
		4	4.0	2.2	39.4	22.1	
		16	8.6	3.4	51.4	20.5	
			Average	4.3		16.6	

A.C. Flexural Stiffness Tests (E,) (after Monismith)

Test Property	Test Temp. (°F)	Number of Mixes	S(X 10^3 psi)		s(X 10^3 psi)		C.V.	
			Average	Range	Average	Range	Average	Range
Flexural Stiffness E.(psi) (Lab Prepared Specimens)	68	14	326.0	190.0-656.0	49.0	4.0-92.0	13.1	2.2-23.8

Flexural Stiffness

Table 9 gives a summary of laboratory variability of the flexural stiffness of asphalt concrete. For a test temperature of 68°F, the mean stiffness value was 326.0×10^3 psi with a range of 190.0×10^3 psi to 656.0×10^3 psi. The standard deviation ranged from 4.0×10^3 psi to 92.0×10^3 psi with an average of 49×10^3 psi while the coefficient of variation ranged from 2.2% to 23.8% with an average of 13.1%.

Modulus values of asphalt concrete used in base materials seem to have more variability than the modulus values of asphalt concrete used in surface courses. Test results from blackbase highway projects (Table 10) show that the mean modulus value ranged from 35×10^3 psi to 91.5×10^3 psi with a weighted average of 58.8×10^3 psi. The coefficients of variation ranged from 25% to 52% with a weighted average of 40%. There is no reference made to the temperature used for these tests. Therefore, since the modulus of elasticity is a function of temperature, further research should be conducted before using the values obtained from this report.

Table 11 is a summary of flexural stiffness measurements on field samples. These values were found by using the pulse load method. For a test temperature of 68°F, mean values of flexural stiffness ranged from 1.34×10^5 psi to 1.79×10^5 psi with an average of 1.58×10^5 psi. The coefficient of variation ranged from 23.5% to 27.6% with an average of 25.0%. For a test temperature of 40°F, the flexural stiffness ranged from 5.90×10^5 psi to 7.12×10^5 psi with an average of 6.71×10^5 psi. The coefficient of variation ranged from 18.8% to 27.2% with an average of 22.4%.

Poisson's Ratio

Kennedy, Hudson, and McCullough (5) investigated the variability of the Poisson's ratio of asphalt concrete. Using the results from 15 specimens, they found that the mean value of was 0.40 and that its coefficient of variation was 27%. In Table 10, the mean value of Poisson's ratio varied from 0.16 to 0.34 with an average of 0.25. The coefficient of variation ranged from 38% to 75% with a weighted average of 52%. There is no reference made as to what temperature that was measured at. Therefore, since is a function of temperature, further research should be conducted before using these values.

CEMENT TREATED MATERIALS

Thickness

For cement-treated bases with a thickness of 4 to 8 inches, the standard deviation ranged from 0.60 to 0.72 inches. The coefficient of variation ranged from 7.5% to 18% (9).

Modulus of Elasticity

There are vary large variations in the modulus of elasticity for cement-treated bases. Table 12 shows that mean values of the modulus ranged from 0.6 x 10^6 psi to 1.90 x 10^6 psi. The standard deviation ranged from 0.36 to 10^6 psi to 1.19 x 10^6 psi. The coefficient of variation ranged from 53% to 83%.

SUBGRADE

Modulus of Subgrade Reaction

Treybig, Hudson, and McCullough (8) analyzed data from the AASHO Road Test. This subgrade material had an average modulus of subgrade reaction of 100 pci and a coefficient of variation of 16%. This

TABLE 10. Summary of Test Results for Cored Specimens from Blackbase Highway Projects in Texas (Kennedy)

Project Identification	Number of Specimens		Distance Covered (miles)	Indirect Modulus of Elasticity Mean (10^3 psi)	Indirect Modulus of Elasticity CV (%)	Indirect Poisson's Ratio Mean	Indirect Poisson's Ratio CV (%)
2-A	76		15.0	38.6	32	0.34	39
8-A	16		3.3	91.5	29	0.28	40
13-A	14		8.0	44.9	46	0.16	58
13-B	28		4.3	87.3	62	0.16	73
13-C	16		3.0	35.0	40	0.26	57
15-A	49		10.9	86.1	59	0.23	47
17-B	100		19.1	55.2	44	0.24	41
18-B	12		0.9	42.2	24	0.20	64
19-A	54		19.3	55.2	33	0.32	38
19-B	36		15.2	64.7	34	0.16	67
	Weighted Average			58.8	40	0.25	52
	CV of means (%)			36	--	.28	--
	Variation	Limits		35-91.5	24-62	.16-.34	38-73
		Range		56.5	38	0.18	35

TABLE 11. Flexural Stiffness Measurements on Field Samples of Asphaltic Concrete Using Pulse Loading Method (Kennedy)

Sample Group	No. of Speci-mens	Measured Stiffness (psi x 10^5)					
		68°F			40°F		
		Mean	Std. Dev.	CV (%)	Mean	Std. Dev.	CV (%)
Specimens from Surface Course							
1	19	1.79	0.42	23.5	6.80	1.53	22.5
2	20	1.65	0.39	23.6	7.03	1.91	27.2
3	20	1.52	0.41	26.9	7.12	1.41	19.8
4	19	1.34	0.37	27.6	5.90	1.11	18.8
				25.0			22.4
Lab Compacted	26	1.29	0.22	17.0	5.76	0.74	12.8
Specimens from Base Course							
1	12	1.57	0.42	26.6	5.95	1.54	25.9
2	12	1.39	0.26	18.7	5.66	3.14	55.5
3	8	1.47	0.41	27.9	4.40	0.90	20.4
4	10	1.42	0.42	29.6	4.96	1.22	24.6
				25.3			33.0
Lab Compacted	29	1.19	0.19	16.0	5.31	1.50	28.2

TABLE 12. Summary of Properties for Cement-Treated Bases for Airfield and Highway Pavements (Kennedy)

	Identification		Type of Material	Number of Specimens	Distance Covered (miles)	Indirect Modulus of Elasticity* Mean (10^6 psi)	Indirect Modulus of Elasticity* CV (%)
AIRFIELDS	O'Hare Run-way 4R-22L	Top	-	15		1.9	53
		Bottom		15		1.8	66
		Combined		30		1.8	59
	DFW		Kingburg Gravel	4		-	-
				Weighted Average			
HIGHWAYS		12-A	Sand Shell	32	-	1.76	72
		19-A	Soil Cement	20	1.4	1.05	83
		19-B	Soil Cement	19	1.2	0.73	57
		20-A	Burned Clay	29	1.5	0.60	60
				Weighted Average		0.09	68
				CV of Means(%)		50	-
				Variation	Limits	0.60-1.76	57-83
					Range	1.16	26

*Assumed Poisson's Ratio-0.22

coefficient of variation should only be applicable to a k of 100 pci. Further information should be gathered before drawing any conclusions about the coefficients of variation for other values of k.

Resilient Modulus

The resilient modulus tended to increase with a decrease in deviator stress. Table 13 shows mean modulus values of 12×10^3 psi, 16×10^3 psi, and 19×10^3 psi for deviator stresses of 8 psi, 5 psi, and 2 psi, respectively. The coefficient of variation ranged from 5.3% to 52%. The average standard deviation was 3.75×10^3 psi.

California Bearing Ratio (CBR)

Table 14 shows variability data for the subgrade CBR. Mean CBR values ranged from 4.2 to 26.3. The standard deviation ranged from 0.9 to 8.4 while the coefficient of variation ranged from 17.9% to 36.9%.

TABLE 13. Summary of Resilient Moduli for Subgrade Soils for Airports (Kennedy)

Airport	Project Identification	Number of Tests	Deviator Stress (psi) 8 Mean (10^3 psi)	8 CV (%)	5 Mean (10^3 psi)	5 CV (%)	2 Mean (10^3 psi)	2 CV (%)
Palmdale	Runway 7-25	4	10	52	10	47	8.4	26
		10	24	26	30	29	38	34
O'Hare	Runway 9R-27L	8	7.9	48	-	-	-	-
		3	6.3	21	8.7	12	10	13
	Runway 4R-22L	4	5.7	17	7.0	5.3	8.4	16
Richmond	Taxiway S-4 Runway 2 Taxiway D	3	8.6	8.3	12	20	11	18
Midway	Runway 4R-22L 13R-31L	4	2.4	10	4.0	7.9	5.4	6.6
	Weighted Average		12	29	16	22	19	22
	Variation: Limits		2.4-24	8.3-52	4-30	5.3-47	5.4-3.8	6.6-34
	Variation: Range		21.6	43.7	26	41.7	32.6	27.4

Table 14

Subgrade CBR Variability
(reduced from Yoder and Witczak)

Mean CBR	S	CV(%)	Remarks
7.1	1.6	22.3	(In-situ; compacted subgrade)
4.2	0.9	21.4	(Estimated; after moisture equil)
26.3	8.4	31.9	(In-situ; compacted subgrade)
20.3	7.6	36.9	(Estimated; after moisture equil)
18.2	4.8	26.2	(In-situ; compacted subgrade)
7.8	1.4	17.9	(Undisturbed samples; soaked)

Chapter 3

SUMMARY AND RECOMMENDATIONS

The purpose of this report was to give a general summary of values, or a range of values, that describe material variabilities. It summarizes a state-of-the-art review in variabilities of material properties which are used in pavement design.

If a probabilistic design methodology has been developed, a measure of the variability of the specific parameter in question must be determined. This variability information, however, is not always readily available. This report, then, is to be used as a guide in determining the level of variability to be used for a design parameter. Table 15 is a table of variability recommendations which may be used for different levels of inherent variability. As variability data becomes available, it should be used to upgrade or to check Table 15. If variability information pertaining to a particular project is available, it should be used in lieu of the information from Table 15.

Table 15. Variability Recommendations

	Coefficient of Variation (%) Inherent Variability		
	Low	Average	High
PCC MATERIALS			
Thickness	1-3	4-6	7-9+
Modulus of Elasticity	20-30	30-40	40-50+
Poisson's Ratio	8-12	13-16	17-20+
Modulus of Rupture	10-13	14-17	18-20+
ASPHALT CONCRETE MATERIALS			
Thickness	1-5	5-10	10-15
Dynamic Modulus			
Temp: 40°F	8-10	11-13	14-16+
70°F	10-12	13-15	16-19+
100°F	18-20	21-22	23-24+
Base Modulus of Elasticity	25-35	35-45	45-55+
Flexural Stiffness			
Temp: 40°F	15-20	20-25	25-28+
68°F	20-23	24-26	27-30+
Poisson's Ratio*	35-48	49-62	63-75+
CEMENT-TREATED MATERIALS			
Thickness	6-10	11-15	16-19+
Modulus of Elasticity	53-63	63-73	73-83
SUBGRADE MATERIALS			
Modulus of Subgrade Reaction*	10-20	20-35	35-50+
Resilient Modulus	10-20	20-35	35-50+
CBR	15-22	23-31	32-40+

* See Text for additional information

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VOLUME II

MATHEMATICAL FORMULATION OF RELIABILITY MODELS UTILIZED IN RIGID PAVEMENT STUDIES

TABLE OF CONTENTS

LIST OF FIGURES

Chapter 1

INTRODUCTION

For the past several years, probabilistic/reliability analyses have slowly been introduced in pavement design schemes to account for the known variation of the salient design parameters. In existing deterministic design methods, material variability effects on pavement performance may be only indirectly taken into account. Frequently, the design parameters (unique values) are determined from either laboratory or field results. If multiple values are determined, the engineer may select (from the distribution of values) a design percentile level (e.g. . . . 85th) for use in the deterministic solution. The confidence level adopted and reflected by the percentile value is currently based on the engineering judgment and experience of the designer. This approach, however, cannot be easily or directly used to quantitatively evaluate the effect of material variability in design analysis, cost studies or reliability solutions. The probabilistic approach which directly implements the variability distribution of all significant parameters provides the only true, accurate measure of the reliability of the entire pavement system.

The probabilistic analysis of pavement performance is based on concepts and formulations developed in the statistical field dealing with system reliability. This volume presents a summary of

(a) the definitions and mathematical formulation of the general problem applicable for any probability density distribution;

(b) the existing approaches for developing pavement design solutions

in probability and reliability terms. It includes: (i) the interference theory or stress-strength approach, (ii) the simulation technique and (iii) the approximate closed-form probabilistic formulation, based on the Taylor series expansion of the dependent variable.

The various theories are discussed from the viewpoint of their possible implementation in the rigid pavement design procedure of the USACE for airfield systems.

Chapter 2

DEFINITIONS AND MATHEMATICAL FORMULATION

General

This chapter presents a brief overview of basic definitions and theorems related to the probabilistic formulation of design variables applied to the pavement design solution. Further background, details proof of theorem, etc. . . . can be found in most standard textbooks on Probability and Reliability (see, for example, Hines and Montgomery (1))

Probability and Reliability Concepts

Reliability is defined as the probability of success of an event and/or a statement of the error or precision of an estimate. This event, which can take a finite or infinite number of values, is called a random variable. The distribution of these values is related to the probability distribution of the random variable by the concept of probability, i.e.

$$P(X = x_k) = f(x_k) \qquad k = 1, 2, \ldots \tag{2.1}$$

$$\text{with } \int_{-\infty}^{+\infty} f(x_k)\, dx_k = 1$$

where $P(X = x_k)$ represents the probability that the random variable takes the value x_k, and $f(x_k)$ is the density probability distribution of X at x_k. This can also be expressed through the cumulative distribution function defined as

$$P(X \leq x_k) = F(x_k) = \int_{-\infty}^{x_k} f(u)\, du \tag{2.2}$$

When success is defined by X taking all values greater than x_k, reliability takes the form of

$$R = P(X > x_k) = 1 - F(x_k) = 1 - \int_{-\infty}^{x_k} f(u)\, du \qquad (2.3)$$

It can be seen that reliability can be directly computed from the probability distribution of the random variable, directly from its cumulative probability distribution, or from integration of its density probability distribution.

Extension to Multi Variate Expressions

The case of one random variable is the simplest solution. However, in practice, two or more random variables are involved in the process. If one examines the case of two random variables, the joint distribution function of the random variables X and Y is defined as:

$$P(X = x_j,\ Y = y_k) = f(x_j,\ y_k) \qquad (2.4)$$

$$\text{with } \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x_j,\ y_k)\, dx_j\, dy_k = 1$$

The above probability expresses the occurence of two events which can be either dependent or independent. In the case of independent variables, the "product rule" applies, i.e.

$$P(X = x_j,\ Y = y_k) = [P(X = x_j)] \cdot [P(Y = y_k)] \qquad (2.5)$$

$$\text{or} \quad f(x,\ y) = [f_1(x)] \cdot [f_2(y)]$$

where $f_1(x)$ and $f_2(y)$ are called the marginal probabilities of X and Y. The extension of the one random variable to two or more random variables is easily done:

$$P(X \leq x_j, Y \leq y_k) = [P(X \leq x_j)] \cdot [P(Y \leq y_k)] \tag{2.6}$$

$$= [F_1(x)] \cdot [F_2(y)]$$

However, when the random variables are dependent, the product rule does not hold and it is replaced by the so-called conditional probability which is expressed by the following:

$$P(Y = y_k / X = x_j) = \frac{P(Y = y_k, X = x_j)}{P(X = x_j)} = \frac{f(x_j, y_k)}{f_1(x_j)} \tag{2.7}$$

where

$P(Y = y_k/X = x_j)$ is the probability of $Y = y_k$ given that

X is equal to x_j

The joint probability of two dependent random variables is therefore given by:

$$f(x, y) = [f(y/x)] \cdot [f_1(x)] \tag{2.8}$$

The probability of Y being between the values c and d given that $x < X < x+dx$ is given by:

$$P(c < Y < d / x < X < x+dx) = \int_c^d f(y/x)\, dy \tag{2.9}$$

This can be extended for the whole range of X values to bring the marginal distribution for Y as follows:

$$P(Y \leq d / -\infty < X < +\infty) = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{d} f(y/x) dy \right] f_1(x) dx \tag{2.10}$$

$$= \int_{x=-\infty}^{+\infty} \int_{y=-\infty}^{d} f(x,y)\, dx\, dy$$

Change of Variables

If X is a continuous random variable with a probability density function f(x), $U = \phi(X)$ and $X = \Psi(U)$; then the probability density of U is given by g(u) where

$$g(u) = f(x)\left|\frac{dx}{du}\right| = f(\psi(u))|\ \psi'(u)| \qquad (2.11)$$

If X and Y are continuous random variables having a joint density probability function f(x,y), $U = \phi_1(X,Y)$, $V = \phi_2(X,Y)$, $X = \psi_1(U,V)$ and $Y = \psi_2(U,V)$; then the joint density function of U and V is given by g(u,v) where

$$g(u,v) = f(x,y)\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = f[\psi_1(u,v),\ \psi_2(u,v)]\cdot\ |J|$$

where

J = the Jacobian of the transformation (2.12)

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

Probability Distributions of Functions of Random Variables

If X and Y are continuous random variables and $U = \phi_1(X,Y)$, then the density function of U is the marginal density obtained from the joint density of U and V (while V is chosen arbitrarily, V = X or V = Y).

If f(x,y) is the joint density for X and Y, then the density function g(u) of the random variable $U = \phi_1(X,Y)$ is found by differentiating with respect to u the distribution function given by

$$G(u) = P[\phi_1(x,y) \leq u] = \int\int_R f(x,y)\ dx\ dy \qquad (2.13)$$

where R is the region for which $\phi_1(x,y) \leq u$

The density function of the sum of two continuous random variables X and Y, i.e. of $U = X+Y$, having joint density function $f(x,y)$ is given by:

$$g(u) = \int_{-\infty}^{+\infty} f(x, u-x)\, dx \tag{2.14}$$

For the special case where X and Y are independent,

$$f(x,y) = [f_1(x)] \cdot [f_2(y)] \tag{2.15}$$

$$\text{and} \quad g(u) = \int_{-\infty}^{+\infty} [f_1(x)] \cdot [f_2(u-x)\, dx] = f_1 * f_2$$

This is called the convolution of f_1 and f_2.

Expectation

For a continuous random variable X having a density function $f(x)$, the expectation of X is defined as

$$E[X] = \int_{-\infty}^{+\infty} xf(x)\, dx \tag{2.16}$$

The expection of X is very often called the mean of X (denoted by μ_X) and is a measure of central tendency.

If X is a random variable then $Y = g(X)$, a function of X is also a random variable. The expectation of Y is

$$E[Y] = E[g(X)] = \int_{-\infty}^{+\infty} g(x) \cdot f(x)\, dx \tag{2.17}$$

For a function of two random variables $g(X,Y)$

$$E[g(X,Y)] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(x,y) \cdot f(x,y)\, dx\, dy \tag{2.18}$$

The variance of a random variable is defined by:

$$VAR[X] = E[(x - \mu)^2] \tag{2.19}$$

where $g(X) = (x-\mu)^2$ is a function of X. According to the above:

$$E[g(X)] = E[(X-\mu)^2] = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\, dx \qquad (2.20)$$

The variance (or its square root, the standard deviation) is a measure of the dispersion or scatter of the values of the random variable. The covariance of two random variables is defined similarly as:

$$\begin{aligned} \mathrm{Cov}[X,Y] &= E[(X-\mu_x)(Y-\mu_y)] \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (x-\mu_x)(y-\mu_y)\, f(x,y)\, dx\, dy \end{aligned} \qquad (2.21)$$

which is equal to zero when the random variables are independent. When X and Y are dependent, the following relation between the covariances, the variances and the correlation coefficient can be found by:

$$\begin{aligned} \mathrm{Cov}[X,Y] &= \rho\sqrt{\mathrm{Var}[X] \cdot \mathrm{Var}[Y]} \\ \sigma_{xy} &= \rho\sigma_x \sigma_y \end{aligned} \qquad (2.22)$$

where ρ is the correlation coefficient.

Higher orders of expectation of the random variable are defined in the literature, to describe mathematically the probability density distributions of the random variable (skewness, kurtosis, etc.)

Chapter 3

PROBABILISTIC THEORIES FOR PAVEMENT DESIGN

General Background

The reliability of a system composed of several units depends upon the reliability of these units and upon the way they are interconnected (in series or in parallel as defined in the reliability theory). The reliability of each unit can be determined by experiment or assessed from the stress-strength method.

At present, the rigid pavement design framework is based on slab failure only, without any consideration of subgrade or joint failure. It is therefore a one unit system with a large number of variables (traffic, material characteristics and slab geometry). A basic approach to the problem would be that of the stress-strength analytical methodology. The second one would be based on simulation, to bypass the complexity of the basic approach, and the third one would be an approximate one based on normal distribution assumption and Taylor series expansion of the functions. These approaches are discussed in detail.

Stress-Strength Approach (Interference Theory)

The unit is assumed to fail where the "stress"(Y) reaches its ultimate value, its "strength" (X). The probability density distribution of both the stress and the strength must be known. If it is assumed that the stress and the strength are independent, the probability of failure is expressed as the probability of X being lower or equal to a given value of Y, i.e.,

$$P\ (X<y/\ Y=y) = \int_{0}^{y} f_1(x)\ dx \qquad (3.1)$$

Multiplying by the probability that Y is in the neighborhood of y, one obtains the joint probability function

$$P\ (X \leq y,\ y < Y \leq y + dy) \;=\; \int_{-\infty}^{y} f_1(x)\ f_2(y)\ dx \tag{3.2}$$

and extending this to the whole range of Y,

$$P\ (X \leq Y) \;=\; \int_{-\infty}^{+\infty} \int_{-\infty}^{y} f_1(x) \cdot f_2\ (y)\ dx\ dy \tag{3.3}$$

The reliability of the unit is given by

$$\begin{aligned} R = 1 - P(X \leq Y) &= 1 - \int_{-\infty}^{+\infty} \int_{-\infty}^{y} f_1(x)\ f_2(y)\ dx\ dy \\ &= 1 - \int_{-\infty}^{+\infty} F(y)\ f_2(y)\ dy \end{aligned} \tag{3.4}$$

where F(y) is the cumulative probability function of x evaluated at y.

It is seen that the reliability can be computed if the density probability functions of the stress and the strength are known. However, only for very simple cases can the above integral be evaluated in a closed form model. Yet the stress itself can be a function of random variables, and its density probability function can be expressed by the above theorem by differentiating

$$G(u) \;=\; \iint_R f(x,y)\ dx\ dy \tag{3.5}$$

with respect to u.

The problem involving a number of variables is found to be very complex, first to define the probability density distribution of the "stress" and of the "strength", and second, to evaluate the joint probability (or the probability of failure). It should be noted that for the case of the normal density distribution of the stress and the

strength, the probability of failure P(X_Y) is also normally distributed (2). In passing, it should also be noted that log-normal distributions or other statistical distributions (e.g. Beta) can be utilized in the above noted procedure.

Simulation Technique

The technique of simulation has become widely used for estimating distribution parameters of the variable and for providing insight into the cause and effect relationship within a system. In simulation, the system is divided into elements whose behavior can be predicted, at least in terms of probability distributions. They are then combined in their natural order, allowing the computer to present the effect of their interaction on each other. After constructing the model, it is activated by generating appropriate input data to simulate the actual aggregate behavior of the system over time.

The technique of simulation has not been applied widely in estimating pavement performance. Few studies (e.g., Ullitz (3)have advanced reliability based pavement performance schemes. Simulation can be applied for the rigid pavement case to estimate the probability distribution of the overall behavior (performance) of the pavement, given the probability distributions of the individual variables and to estimate the effect of each variable on the overall (aggregate) behavior. Formulation of the model is quite simple and requires the following steps:

(1) Define the random variables and their probability distributions;

(2) Express the rules (equations) that link the different variables to the performance variable - the one whose average value and probability distribution is of interest.

Activation of the model is performed by generating input of the random variables from their probability distribution and deriving the

performance variable. Repeating the process a sufficient number of times gives the description of the probability distribution of the output variable. The appropriate flow chart for simulating rigid pavement performance is shown in Fig. 3.1.

For the case of several variables, and in order to estimate the cause and effect relationships, a factorial simulation experiment must be conducted. The factorial experiment must be designed in such a way to allow to extract the individual effects of the random variables.

It should be noted the number of solutions required (N - in Fig. 3.1) may be quite large in order to accurately reproduce probability distributions of the various variables. Because of this, the simulation technique should be restricted to situations that are too complex to be handled by closed form formulation and/or for cases where there is strong evidence that probability distribution of variables are far from the normal approximation.

Approximate Closed Form Probabilistic Formulation

According to the probability concepts briefly summarized above, the expected value (mean) of a function of variables is expressed as:

$$E\,[g(X_i)] = \int_{-\infty}^{+\infty} g(x_i) \cdot f(x_1,\ldots,x_n)\, dx_1\ldots dx_n \tag{3.6}$$

where $f(x_1, \ldots, x_n)$ is the joint probability distribution of all random variables X_i. This is rather difficult to evaluate. Instead of evaluating integrals, the function may be expressed in terms of the Taylor series expansion evaluated near the variable means and its expected value derived:

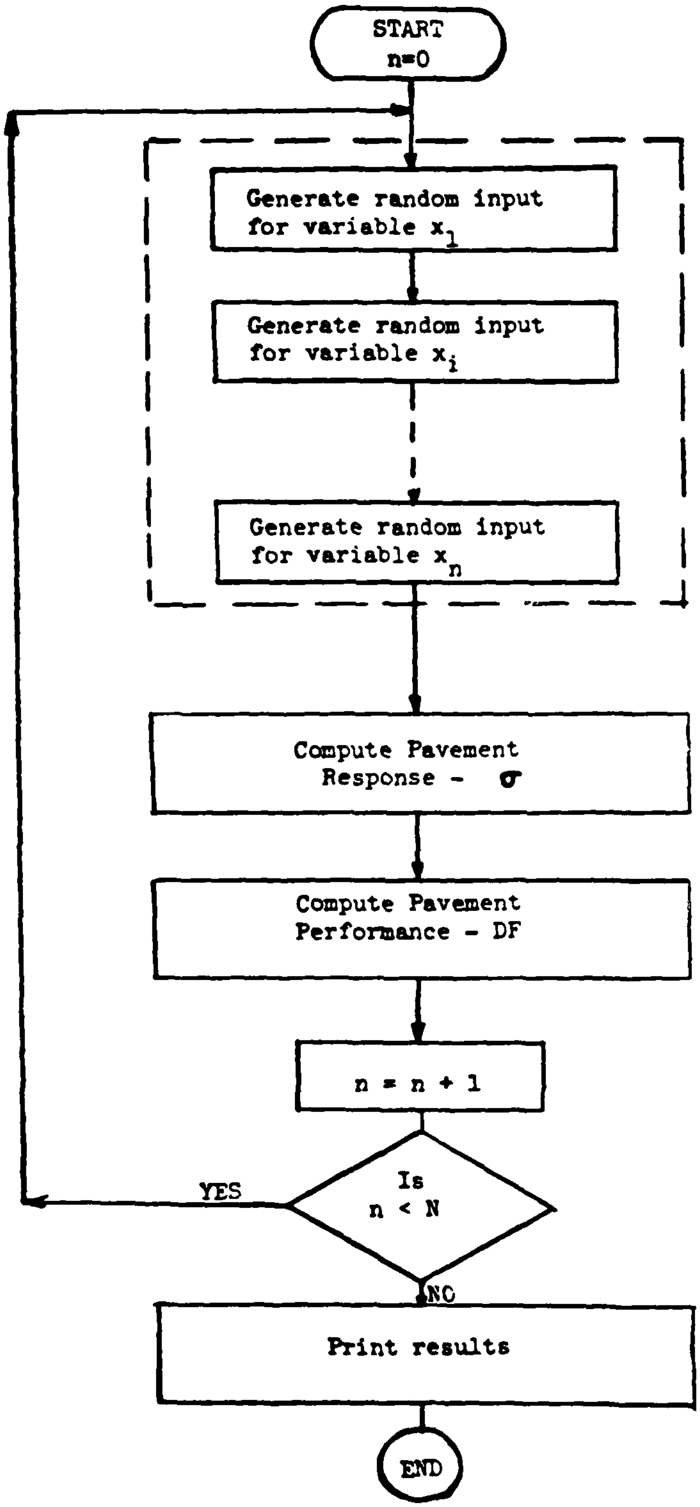

Fig. 3.1 SIMULATION SCHEME

$$g(X_i) = g(X_i)\Big|_{\overline{X}_i} + \sum_{i=1}^{n} (X_i - \overline{X}_i) \frac{\partial g(X_i)}{\partial X_i}\Big|_{\overline{X}_i} + \sum_{i=1}^{n} \frac{(X_i - \overline{X}_i)^2}{2} \frac{\partial^2 g(X_i)}{\partial X_i^2}\Big|_{\overline{X}_i}$$

$$+ \sum_{i=1}^{n} \sum_{\substack{k=1 \\ k \neq i}}^{n} (X_i - \overline{X}_i)(X_k - \overline{X}_k) \frac{\partial^2 g(X_i)}{\partial X_i \, \partial X_k}\Big|_{\overline{X}_i, \overline{X}_k} + R \qquad (3.7)$$

where R is the residual.

In the case of the first-order Taylor series approximation (corresponding to linear approximation), the following can be found.

$$E[g(X_i)] = g(\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_n)$$

$$\text{Var}[g(X_i)] = \sum_{i=1}^{n} \left(\frac{\partial g(X_i)}{\partial X_i}\Big|_{\overline{X}_i} \right)^2 \cdot \text{Var}[X_i] + \qquad (3.8)$$

$$\sum_{i=1}^{n} \sum_{\substack{k=1 \\ k \neq i}}^{n} \left(\frac{\partial g(X_i)}{\partial X_i}\Big|_{\overline{X}_i} \right) \left(\frac{\partial g(X_k)}{\partial X_k}\Big|_{\overline{X}_k} \right) \cdot \text{Cov}[X_i, X_k]$$

where:

$g(\overline{X}_1, \overline{X}_2, \ldots \overline{X}_n)$ is the function $g(X_i)$ evaluated at $\overline{X}_i$ = mean values of the variables X_i

$\frac{\partial g(X_i)}{\partial X_i}\Big|_{\overline{X}_i}$	is the derivative of $g(X_i)$ with respect to X_i, evaluated at $X_i = \overline{X}_i$
$\text{Var}[X_i]$	is the variance of variable X_i
$\text{Cov}[X_i, X_k]$	is the covariance of variables X_i and X_k.

In the case of two variables, it can be shown that:

$$E[g(x,y)] = g(\bar{x}, \bar{y})$$

$$Var[g(x,y)] = \left(\frac{\partial g}{\partial x}\Big|_{\bar{x}}\right)^2 Var[x] + \left(\frac{\partial g}{\partial y}\Big|_{\bar{y}}\right)^2 Var[y] + \qquad (3.9)$$

$$2 \left(\frac{\partial g}{\partial x}\Big|_{\bar{x}}\right) \left(\frac{\partial g}{\partial y}\Big|_{\bar{y}}\right) Cov\ (x,y)$$

It should be noted that the first order approximations of the Taylor series is in most engineering cases a good approximation. Taking the second-order approximation leads to the following mean:

$$E[g(X_i)] = g(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n) +$$

$$\sum_{i=1}^{n} \frac{1}{2} \cdot \frac{\partial^2 g(X_i)}{\partial x_i^2}\Big|_{\bar{X}_i} \cdot Var[X_i] +$$

$$\sum_{i=1}^{n} \sum_{\substack{k=1 \\ k \neq i}}^{n} \frac{\partial^2 g(X_i)}{\partial X_i \, \partial X_k}\Big|_{\bar{X}_i, \bar{X}_k} Cov[X_i, X_k] + R \qquad (3.10)$$

Since $Var[X_i]$ and $Cov[X_i, X_k]$ are relatively small, the approximation is a good one where the second order derivatives of $g(X_i)$ evaluated at $\bar{X}_i$ are also relatively small so that their product will be negligible. The variance term of the second-order approximation involves higher order of measures of central tendency (the skewness and kurtosis) and multiplication of the variances and covariances of the variables. As such, the precise

derivation becomes rather complex. The linear approximation is improved in symmetric probability distributions when the skewness coefficient is nil. Therefore, the approximate closed form formulation should be adopted only when the output variable has a symmetric density distribution. For the closed-form approximation to be appropriate, it should satisfy the following conditions: (a) the output variable is normally distributed and (b) the second order terms of the Taylor series expansion can be neglected, in comparison of the first order form.

Chapter 4

SUMMARY

Three basic approaches for expressing rigid pavement design in probabilistic and reliability terms were summarized. From the above, it is noted that

(a) in the stress-strength approach, definition and characterization of the stress component need to be clarified. In the current design method, initial failure is defined as the "point at which the crack, which originates at the bottom of the slab and migrates upward to the surface, commences to spall and ravel, which produces debris on the surface (4). The "stress" that would cause such a deterioration (cracking initiation and propagation, spalling and ravelling) cannot at this stage be given a mechanistic interpretation.

The stress-strength approach assumes that the variables are independent. Moreover, it is based on comparing the values of the random variables, X - the stress, and Y - the strength, which is equivalent to defining the new variable Z = X-Y. The approach currently followed in the rigid pavement design is quite different. The failure is defined through the Design Factor which is the ratio of the strength (the modulus of rupture) and the stress (further reduced by the load transfer coefficient). For this case, the probability distribution of the design factor remains unknown and quite difficult to determine.

While the discussion has denoted the random variables in authentic fasion, it should be understood that other distributions (i.e., log normal, Beta, Gamma) can be assumed to represent the PDF of each variable. Such assumptions would cause further mathematical problems in the "Strength-Stress" approach to the reliability solution.

(b) Simulation techniques are powerful for solving complex problems, which cannot be solved in closed form. However, full factorial experiment analysis may be very computer time consuming, and hence costly. It could be combined with the approximate closed form solution which could be used for evaluating the effect of each variable on the overall aggregate performance.

(c) Where analytical expressions are available, or can be developed by statistical regression predictive techniques, the direct application of the Taylor Series expansion (approximate closed-form) to a probabilistic formulation of reliability analysis appears warranted. It is without question a very powerful mathematical tool that has great potential in reliability studies of pavement performance.

Because of these general considerations, it is noted that the primary method of developing the probabilistic solutions to the rigid pavement performance analysis of the USACE for airfield pavements was the Approximate Closed-Form solution (Taylor Series). This approach was used to formulate both the probabilistic Westergaard-USACE procedure shown in Volume III as well as the Multi-Layered Elastic Design (developed by the USACE) presented in Volume IV. In the latter case, a simulation scheme is also presented to clearly demonstrate that the two approaches are, from an engineering viewpoint, equivalent. Such a conclusion only enhances the utilization of the Taylor Series approach to the reliability problem. Inherent within this suggested probabilistic approach, is the necessity to develop closed-form solutions and models by statistical regression techniques for various variables important in the design/analysis solution. The development of these particular expressions are clearly presented in the appropriate report Volumes.

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VOLUME III

PROBABILISTIC ANALYSIS OF RIGID AIRFIELD DESIGN
BY THE WESTERGAARD FREE EDGE THEORY

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1

INTRODUCTION

General

The design of rigid pavements (plain and reinforced) for military purposes is currently based upon the classical Westergaard free edge stress slab theory. Present U.S. Army Technical Manuals and U.S. Air Force Manuals use this theory as the basis for their design methods. The Westergaard model consists of a linear elastic slab supported by a dense liquid (equivalent to independent linear springs). While Westergaard developed stress solutions for loads placed at various slab locations (i.e., edge, corner and interior), the military design approach is based upon a (modified) free edge stress condition. (1)

At present, the design method approach is deterministic, i.e., a unique pavement system is designed for the set of input variables which are also unique. The design corresponds to an acceptable performance level (first crack) with an unknown margin of safety. The deterministic design approach, indirectly and only qualitatively, may account for the effect of material variability on pavement performance by judicious selection of the design input values. However, a quantification of these effects is possible and will improve the design procedure by showing the partial effect of each variable.

Study Objective

The aim of this project was to include the design parameter variability in the current USACE design procedure for unreinforced concrete pavements, and to evaluate (quantitatively) their effect on the final design. The final rigid pavement design is then expressed in reliability terms. The

mathematical and analytical development of the probabilistic approach used in this volume is presented in Volume II of the study.

It was also the aim of this project to expand the current design procedure so that systems with more than one foundation layer could be evaluated. This was accomplished through the use of the composite modulus of reaction which considers all layers below the pavement to respond as an "equivalent", "lumped" or "composite" foundation. The layer parameters of this composite section are then used as the foundation material response within the Westergaard model.

Report Organization

This volume has been subdivided into two major chapters. They are:

Chapter 2: Composite Modulus of Subgrade Reaction

Chapter 3: Probabilistic Analysis of PCC Airfield Design

Chapter 2 will discuss the background of the composite modulus of subgrade reaction as well as the development of the equations which are used within the reliability based design method.

Chapter 3 deals with the development of the equation used to predict the maximum stress computations. These equations, in turn, are used to evaluate the variability of the pavement design. The probabilistic solution used in the Westergaard analysis is based upon the Approximate Closed-form Probabilistic Formulation (Taylor Series) discussed in Volume II. The detailed mathematical derivations used to develop this solution are contained in this volume. A computer program has been developed to solve this probabilistic methodology. A user's guide and a program listing have been included in the Appendices.

Chapter 2

COMPOSITE MODULUS OF SUBGRADE REACTION

Introduction

The composite modulus of subgrade reaction was introduced to take into account the combined effect of a layered system beneath the rigid pavement. This composite modulus is a very useful and necessary tool in the evaluation of layered systems. The Westergaard theory is based upon a slab resting on a foundation material. As a consequence, multiple foundation layers cannot be solved for directly. However, the composite modulus is a means of making the layered system into a single equivalent layer which may be used within the Westergaard model. The composite modulus is defined as an equivalent modulus that will lead to the same response from the original layered system (in this case: equal deflections of the subsystem below the slab). The major variables in determining the composite modulus are the layer thickness and modulus. This chapter presents the evaluation of the composite modulus of subgrade reaction for the case of a layered system. All of the composite modulus relationships used in this solution have been based upon those contained in TM 5-824-3 (AFM 88-6, Chap. 3), Rigid Pavements for Airfields Other Than Army, August 1979.

Current USACE Method for Determining the Composite Modulus of Subgrade Reaction (k_c Value)

In the current USACE method (August 1979), several relationships for composite modulus (k_c) are presented for different materials (2). Figure 2.1 shows the k_c relationship for a well-graded crushed material and Figure 2.2 shows the relationship for natural sands and gravels (PI<8). The k_c is developed as a function of the layer thickness, the modulus of

subgrade reaction and the material type. Figure 2.3 shows the k_c relationship for stabilized subbase materials. In this case, the k_c value is dependent on the modulus of elasticity and layer thickness of the stabilized layer as well as the modulus of subgrade reaction.

Figures 2.1 and 2.2 are based on field plate-loading tests while Figure 2.3 is based on computations using the elastic layered theory with corrections (3). These k_c values are based on an equivalent deflection criterion. The deflection of the composite section is equal to the deflection of the layered system under the same 30 inch diameter plate.

Formulation of Composite Modulus Equations

The approximate closed-form probabilistic approach presented in Volume II and implemented in this volume requires the variables to be expressed in equation form. Therefore, the results of the composite modulus from figures 2.1 to 2.3 were analyzed to derive regression equations. This was accomplished by the use of the Statistical Package for the Social Sciences (SPSS). A multiple regression analysis was performed on the information taken from these figures to generate the equations. (The SPSS outputs may be found in Appendix I). The final equations selected resulted in excellent correlation coefficients and agreement between actual and predicted values. The SPSS generated equations are:

For well-graded crushed materials:

$$\begin{aligned} \log k_c = & -1.251182 + 2.219732 \log k_{sg} - 0.2949522 (\log k_{sg})^2 \\ & + 0.08901252h - 0.0004425194\, h^2 \\ & - 0.02901488\, h \log k_{sg} \end{aligned} \tag{2.1}$$

$R^2 = 0.998$, Standard Error of Estimation SEE = 3%

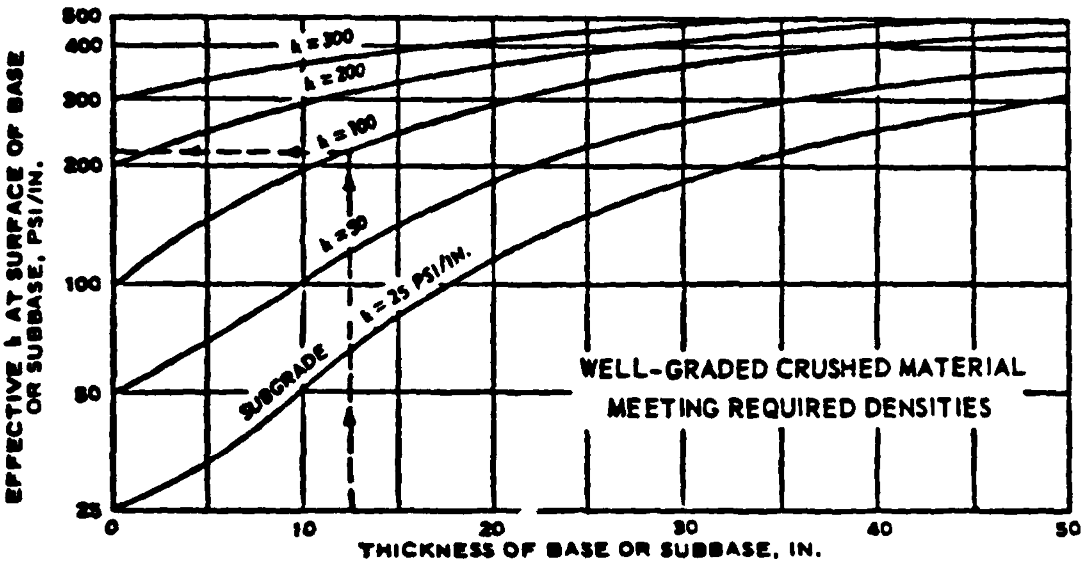

Figure 2.1 COMPOSITE SUBGRADE MODULUS OF REACTION

FOR WELL-GRADED CRUSHED MATERIAL

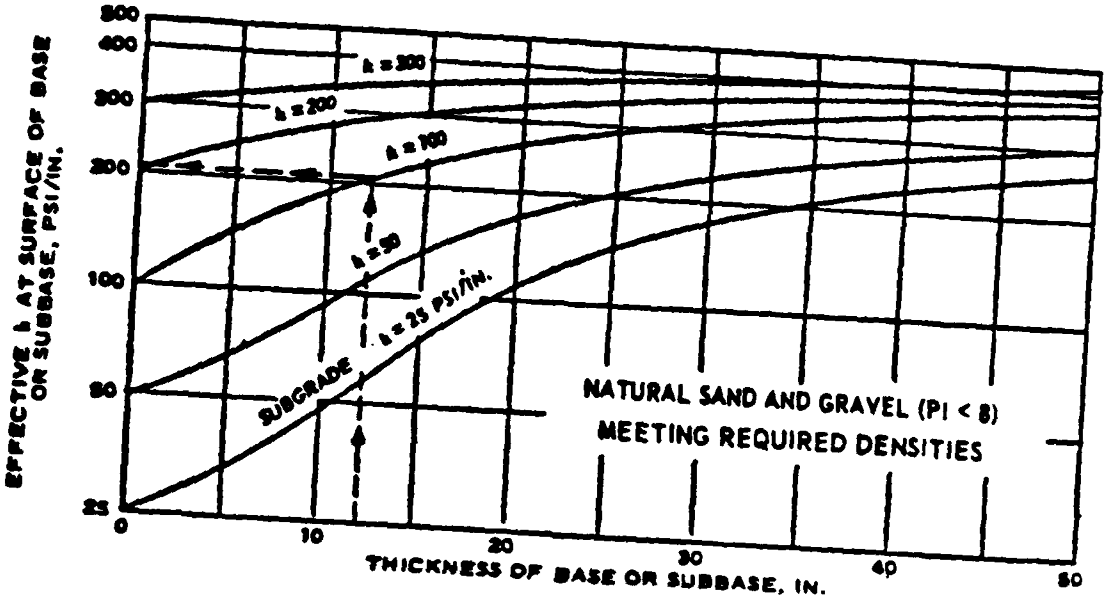

Figure 2.2 COMPOSITE SUBGRADE MODULUS OF REACTION FOR NATURAL SAND AND GRAVEL

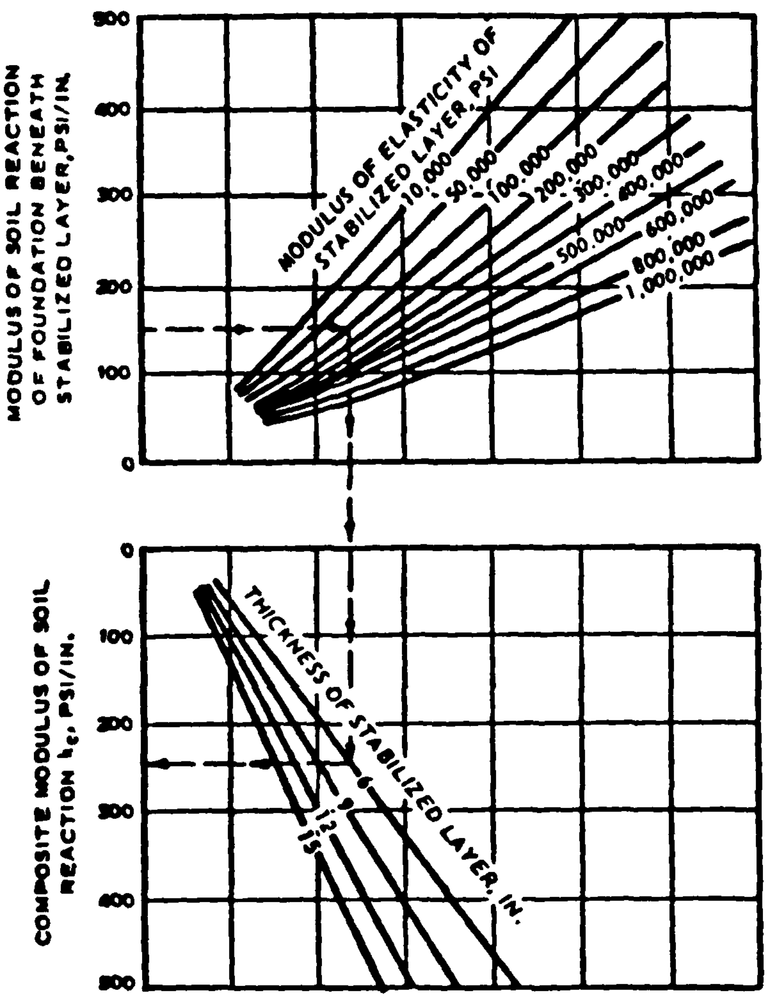

Figure 2.3 COMPOSITE SUBGRADE MODULUS OF REACTION FOR DIFFERENT BASE-SUBBASE MATERIALS

where: h = layer thickness, in.

k_{sg} = modulus of subgrade reaction, pci

For natural sands and gravels (PI<8):

$$\log k_c = -1.296084 + 2.263407 \log k_{sg} - 0.3013741 (\log k_{sg})^2 + 0.08554373h - 0.0002574619 h^2 - 0.03050173 h \log k_{sg} \tag{2.2}$$

$R^2 = 0.997$ SSE = 3.6%

For stabilized layers:

$$\log k_c = -0.1578667 + 1.02813 \log k_{sg} + 0.0544761h - 0.8473852\times10^{-3}h^2 + 0.7254749\times10^{-6}E - 0.1937293\times10^{-12}E^2 - 0.4409096\times10^{-2}h \log k_{sg} - 0.4601653\times10^{-7}E (\log k_{sg}) - 0.2465638\times10^{-8}Eh \tag{2.3}$$

$R^2 = 0.997$ SSE = 2.3%

where: E = modulus of elasticity of the stabilized layer, psi.

It should be stressed that equation 2.3 is only valid within the range of $E = 10^5$ to 2×10^6 psi.

The approximate closed form solution was then used with these equations in order to generate an equation for the $CV^2(k)$ value to be used in the probabilistic analysis. (This value will be discussed in Chapter 3.)

It should be noted that the maximum number of layers for which the program may be used is four (including the slab and the subgrade). The computation of the k_c for a single base/subbase layer is a straightforward solution. The required values (subgrade modulus, layer thickness and the modulus of elasticity, if applicable) are input into the equation for the

composite modulus. If there are two base/subbase layers, the procedure used is as follows:

a. The single layer procedure is followed for the subgrade and the layer directly above it to obtain a k_c' value

b. This k_c' value is then used as the foundation material of the upper base/subbase layer

c. Step (a) is repeated to find a k_c for the entire subsystem.

Since extrapolated values were used in the generation of equation 2.3, it is possible to get a composite modulus greater than the maximum k_c for design of 500 pci. In the program, however, a maximum value of 500 pci is used. The actual calculated k_c value is printed as output along with an indication that the maximum design k is being used. These equations have been set up for a pavement system which has an increasing modulus from the subgrade. Therefore, when analyzing two base/subbase layers, the stiffer layer must be on top.

There are two approaches available for determining the composite modulus of subgrade reaction for stabilized layers. One method accounts for the base material by increasing the modulus of the subgrade while the other method accounts for the base by using a section of PCC with an increased thickness.

For the first method, the composite k-value chart (fig. 2.3) was based on calculations using the elastic layer theory. The values which were obtained by using the elastic layer theory were then corrected based on field data and experience. The value of k_c was determined by using equivalent deflections. The k_{sg} value was raised until the deflection of the "improved" subgrade equalled that of the base-subgrade system.

The other method of accounting for a base material is the use of an increased PCC thickness. This increased thickness is determined by the partially bonded rigid overlay pavement design equation:

$$h_{doc} = \sqrt[1.4]{h_{dc}^{\;1.4} - (0.0063\ \sqrt[3]{E_{fc}}\ h_b)^{1.4}}$$

This equation is based upon equivalent stiffnesses ($D = \frac{Eh^3}{12(1-\mu^2)}$) i.e., the stiffness of a PCC layer for a given modulus of elasticity and thickness is equivalent to the stiffness of a base layer for a given modulus of elasticity and thickness. From this, the increased thickness of the PCC layer may be calculated.

In this probabilistic design methodology, the method of the composite modulus of subgrade reaction has been used. It has been chosen because it yields the best estimate of values to be used in a composite pavement section.

Chapter 3

PROBABILISTIC ANALYSIS OF PCC AIRFIELD DESIGN USING THE WESTERGAARD FREE EDGE THEORY

Introduction

The analysis presented in this chapter is based on the approximate closed-form probabilistic approach which was formulated in Volume II. The use of this closed-form approach requires stress computations which can be made with the H-51 computer program. While the use of the H-51 is justified for the pavement design for a particular aircraft, it can become uneconomical for the probabilistic approach due to the large amount of computer time needed. The analysis was therefore confined to the controlling aircraft of each USAF AGI group. Solutions for other aircraft may be developed by using the user defined option in the program to develop the regression constants to be used in the maximum tensile stress equation.

The probabilistic approach presented in the following paragraphs includes:

(1) Stress computations and derivation of an equation to predict the maximum tensile stress for USAF AGI 1 to USAF AGI 15;

(2) Derivation of the relationship between variances of the dependent and independent variables for the approximate closed-form approach. The linear or first order Taylor series expansion is assumed as presented in Volume II.

Stress Computation

The Corps of Engineers uses the H-51 computer program (4) to compute the bending stress at the edge of a slab supported by a dense liquid. The program, developed by General Dynamics Co., numerically integrates the number of blocks of the influence chart covered by the load. The wheel

load is assumed to be uniformly distributed over an elliptical area shape with any desired ratio of the axes. The results of the computation are generally within 2-3 percent accuracy.

The numerical computation of the stress constitutes a limitation for deriving a closed form probabilistic model. It was therefore necessary to develop an equation for the stress at the edge for the load of interest. The form of the equation was taken from the original Westergaard's work. Westergaard (4,5,) developed an approximate general formula for stresses for the interior and edge cases for any loaded area. The formula for the edge case reads as follows (6):

$$\sigma_e = \frac{3(1+\mu)P}{\pi(3+\mu)h^2}\left[4K - 0.28 - \frac{4}{3}\mu - \mu^2 + (1-\mu)S + 1.18(1+2\mu)\frac{\bar{y}}{\ell}\right] \quad (3.1)$$

where:

$$K = \frac{-1}{A}\int_A \ln\frac{r}{\ell}\, dA$$

$$S = \frac{-1}{A}\int_A \cos 2\Theta\, dA$$

$$\ell = \sqrt[4]{\frac{E\,h^3}{12(1-\mu^2)k}}$$

σ_e = tensile stress at the bottom of the slab along the edge or joint at x=y=0.

P = load

μ = Poisson's ratio of the concrete

h = thickness of the slab

K and S = area coefficients defined by the equations noted

A = area of contact

r,θ = radius and angle in polar coordinates describing the distance of the infinitesimal area from the origin

ℓ = radius of relative stiffness given by the equation noted

k = modulus of subgrade reaction

$\bar{y}$ = the distance from the edge or joint to the center of gravity of the load.

According to Westergaard's results, Equation 3.1 can be written as follows, for the elliptical contact area:

$$\sigma_e = \frac{3(1+\mu)P}{\pi(3+\mu)h^2} \left[4 \ln\ell + f(x,y) + 1.18(1 + 2\mu)\frac{\bar{y}}{\ell}\right] \qquad (3.2)$$

where f(x,y) is the function of the load configuration and geometry. It should be observed that this function is constant for a given loading (aircraft) condition.

It should be stressed that equation (3.1) was developed using a finite polynomial series and is restricted to the case of a "small distance" of the loaded area to the origin at which the stress is computed. Investigation of the extent of this "small distance" for the particular case of a rectangular loaded area showed that the small distance can be as large as about 1.5 ℓ. Without this restriction, equation (3.2) is a powerful result which simplified the analysis. It is interesting to note that, excluding the load geometry conditions, only two pavement variables: h and ℓ, determine the edge stress.

With the above restriction in mind, the following general equation form for expressing the edge stress as function of h and ℓ was adopted:

$$\sigma_e = \frac{1}{h^2}\left[a_o + a_1 \ln\ell + a_2/\ell\right] \qquad (3.3)$$

where a_o, a_1, a_2 - coefficient dependent upon load, load configuration and geometry.

The steps involved in computing the above coefficients for any aircraft type are as follows:

(1) Compute the maximum stress at the edge induced by load for different ℓ values. This can easily be achieved using the H51 computer program by choosing one set of values of k, E (modulus of reaction of subgrade and modulus of elasticity of concrete) and varying h - the slab thickness.

(2) Find the coefficients of a_0, a_1, a_2 by the least square error method, over the stress values computed in step (1).

The methodology is illustrated for the aircraft designated AGI 1 to AGI 15. Their characteristics are shown in Table 3.1. The results of these computations are shown in Appendix II. The stresses computed using eq. (3.3) are also shown in Appendix II. It can be observed that they are almost identical to the original values computed from the H-51 computer program. In general, correlation coefficients greater than 0.999 were obtained. The errors in computing the stresses with the regression equation have been tabulated as a percentage of the original stress and are also presented in Appendix II. (The maximum error is -0.81 of 1%). These results clearly support the use of equation (3.3) for a closed-form solution of the problem.

If the analysis is to be used for a user defined aircraft (other than AGI type), it is necessary to input at least five slab thickness levels. This is necessary so that the least square error regression has enough data points to predict coefficients which are truly representative of the data.

Table 3.1

Summary of USAF Evaluation
Characteristics for Aircraft Groups

USAF Aircraft Group Index	Controlling Aircraft	Type Gear	Tire Spacing	Tire Contact Area	Max.Gross Weight (kips) Gear	Aircraft
1	C-123	Single Wheel	-	275 in^2	27.0	60.0
2	F-4	Single Wheel	-	100 in^2	27.0	60.0
3	F-111	Single Wheel	-	241 in^2	45.0	100.0
4	C-130	Single Tandem	60"cc	400 in^2	69.75	155.0
5	C-9 (DC-9)	Twin Wheel	26"cc	165 in^2	51.3	108.0
6	T-43 (B-747)	Twin Wheel	30.5"cc	174 in^2	54.625	115.0
7	B-727	Twin Wheel	34.0"cc	237 in^2	90.25	190.0
8	E-3 (B-707)	Twin-Tandem	34.5" x 56.0"cc	218 in^2	159.6	336.0
9	C-141	Twin-Tandem	32.5" x 48.0"cc	208 in^2	155.3	345.0
10	C-5	Twin-Delta-Tandem	34.0" x 53.0"cc x 65.0"cc	285 in^2	361.4	769.0
11	KC-10 (DC-10-30)	Twin Tandem	54.0" x 64.0"cc	294 in^2	212.04	558.0
12	E-4 (B-747)	Twin Tandem	44.0" x 58.0"cc	245 in^2	369.6	778.0
13	B-52	Twin-Twin	37.0" x 62" x 37.0"cc	267 in^2	249.6*	480.0
14	OV-1	Single Wheel	-	70 in^2	8.235	18.3
15	C-54	Dual Wheel	18.0"cc	106 in^2	21.15	47.0

*Gear Load does not include bicycle gear factor

Probabilistic Closed-form Approach

General Formulation

The rigid pavement design is based on computation of the design factor DF defined as:

$$DF = \frac{MR}{\alpha\sigma} \qquad (3.4)$$

where MR = modulus of rupture of concrete

α = load transfer coefficient (=0.75 in the Corps of Engineers procedure)

σ = free edge stress induced by load (Westergaard solution)

The design factor is related to the number of coverages on the basis of USACE test section results and experience. The relationships are shown in Figs. 3-1a to Fig. 3-1c for three levels of failure mode: (i) initial failure, (ii) shattered slab condition and (iii) complete failure. Therefore, knowledge of the design factor and its probability distribution leads to a complete description of the design in probabilistic and reliability terms. In the following, the closed form approach is described.

In equation (3.4) the stress is expressed as function of the material and slab geometry variable, (E, μ , k and h) using equation (3.3). In the analysis, a constant μ of 0.15 was used. It has been held as a constant since there is a relatively insignificant effect of μ on σ. Also, H-51 uses a equal to 0.15 in order to generate the stress predictor coefficients. Therefore, μ is equal to a constant value of 0.15 instead of being stoichastic. Therefore, the average design factor is:

$$\overline{DF} = \frac{\overline{MR}}{\overline{\alpha} \cdot \sigma(\overline{E}, \overline{k}, \overline{h})}$$

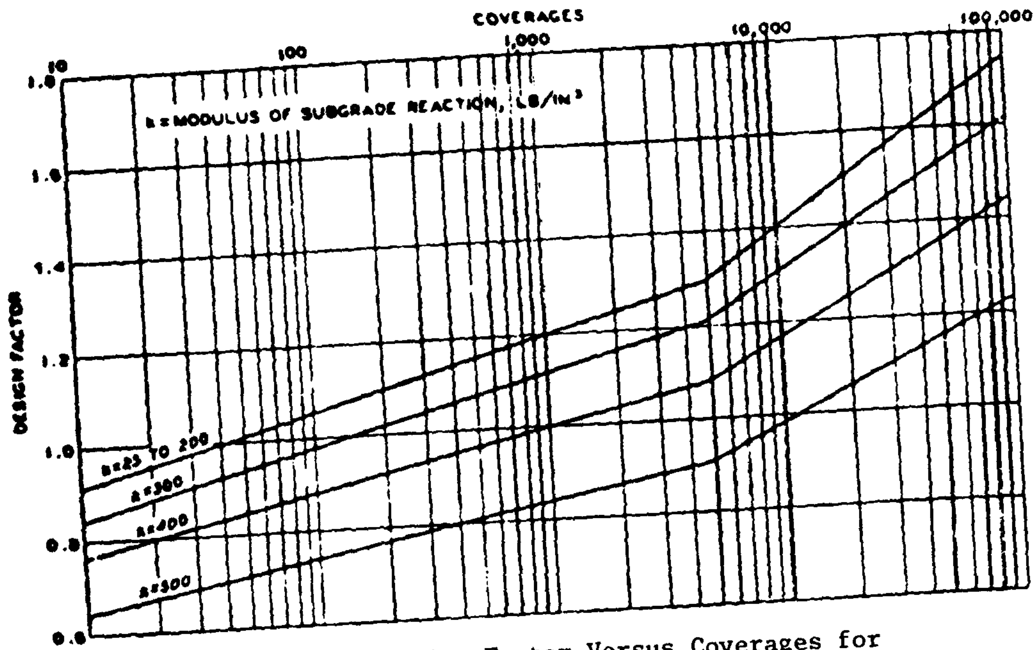

Figure 3.1a Design Factor Versus Coverages for Initial Failure Condition.

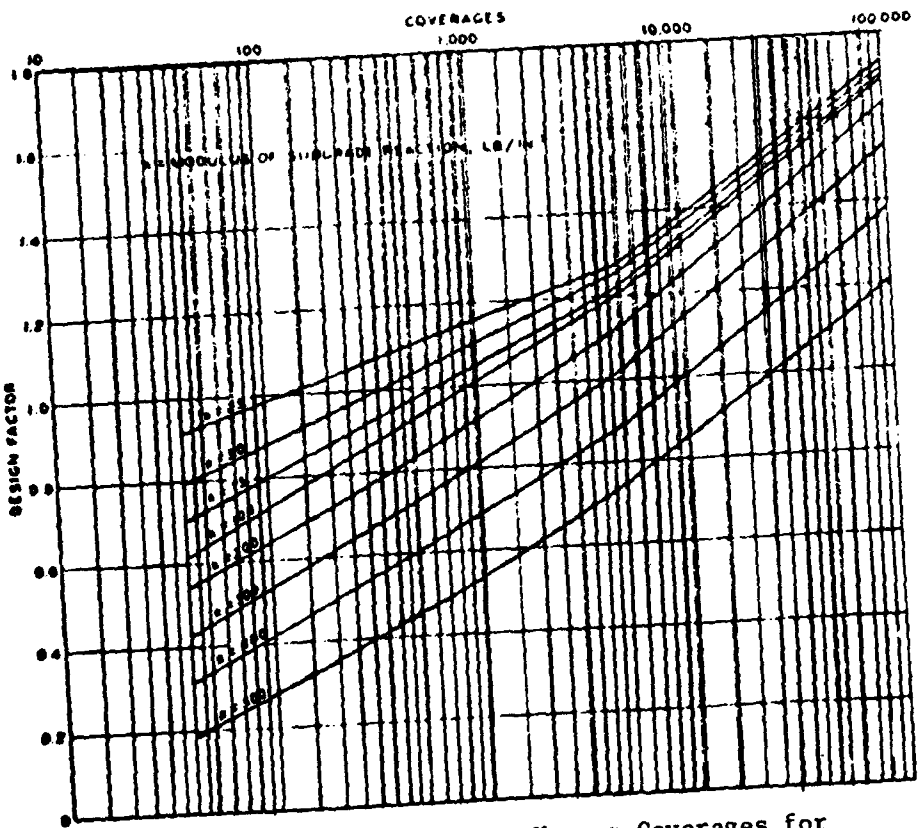

Figure 3.1b Design Factor Versus Coverages for Shattered-Slab Failure.

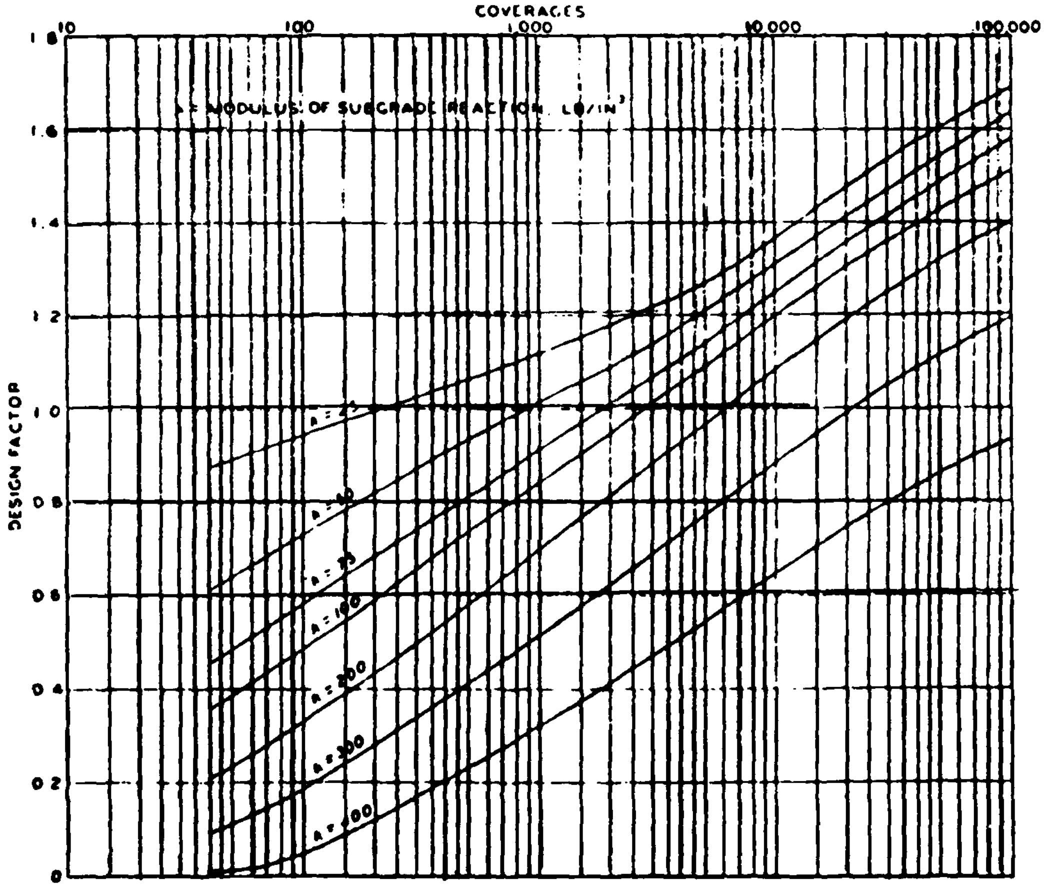

Figure 3.1c Design Factor Versus Coverages for Complete Failure Condition.

and its variance is:

$$\mathrm{Var[DF]} = \sum_{i=1}^{n} \left(\frac{\partial DF}{\partial X_i}\bigg|_{\bar{X}_i}\right)^2 \mathrm{Var}[X_i] +$$

$$\sum_{i=1}^{n} \sum_{\substack{i=1 \\ k \neq 1}}^{n} \left(\frac{\partial DF}{\partial X_i}\bigg|_{\bar{X}_i}\right) \left(\frac{\partial DF}{\partial X_k}\bigg|_{\bar{X}_k}\right) \mathrm{Cov}[X_i, X_k] \qquad (3.6)$$

where X_i - denotes the different variables, MR, α, E, k, h
- $\mathrm{Var}[X_i]$ - variance of variable X_i

Covar $[X_i, X_k]$ - covariance of variables X_i and X_k which is zero when these variables are independent, and equals $\rho \sqrt{\mathrm{Var}[x_i] \cdot \mathrm{Var}[x_k]}$ when the variables are dependent, with a correlation coefficient ρ.

The derivatives of DF with respect to all variables can be found from eq. 3.3 and 3.4 using the chain rule. For example, the derivative of DF with respect to E is given by

$$\frac{\partial DF}{\partial E} = \frac{\partial DF}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial E} = -\frac{MR}{\alpha\sigma^2} \cdot -\frac{\partial \sigma}{\partial E}$$

$$\text{and} \quad \frac{\partial \sigma}{\partial E} = \frac{\partial \sigma}{\partial \ell} \cdot \frac{\partial \ell}{\partial E} = \frac{a_1 - a_2/\ell}{h^2 \ell} \cdot \frac{\ell}{4E}$$

leading to

$$\frac{\partial DF}{\partial E} = -\frac{MR}{\alpha\sigma^2} \cdot \frac{(a_1 - a_2/\ell)}{4h^2 E} \qquad (3.7)$$

Similarly:

$$\frac{\partial DF}{\partial k} = -\frac{MR}{\alpha\sigma^2} \cdot \frac{(a_1 - a_2/\ell)}{4h^2 E}$$

$$\frac{\partial DF}{\partial h} = -\frac{MR}{\alpha\sigma^2} \left[-\frac{2\sigma}{h} + \frac{3(a_1 - a_2/\ell)}{4h^2 \cdot h} \right]$$

and:
$$\frac{\partial DF}{\partial MR} = \frac{1}{\alpha\sigma}$$

$$\frac{\partial DF}{\alpha\sigma} = \frac{MR}{\alpha^2\sigma}$$

For the following, only MR and E are assumed to be dependent. If the coefficient of variation, CV, is defined as the ratio of the standard deviation to the mean value of the variable, then substituting eq (3.7) into eq. (3.6) and rearranging the terms lead to:

$$\begin{aligned} CV^2\,[DF] = {} & CV^2[MR] + CV^2[\alpha] + \\ & \left[\frac{a_1 - a_{2/\ell}}{4\sigma h^2}\right] (CV^2[E] + CV^2[k]) + \\ & \left[-2 + \frac{3(a_1 - a_{2/\ell})}{4\sigma h^2}\right]^2 CV^2[h] + \\ & -2\left[\frac{a_1 - a_{2/\ell}}{4\sigma h^2}\right] \rho[MR,E] \cdot CV[MR] \cdot CV[E] \end{aligned} \tag{3.8}$$

where $CV[X_i]$ is the coefficient of variation of variable X_i. In this equation, the value of $CV^2[k]$ is either the input value of $CV^2[k]$ or it is the calculated value of $CV^2[k]$ if the composite modulus of subgrade reaction is calculated. These $CV^2[k]$ values may be calculated for the materials found in equations 2.1 to 2.3 by using the first-order Taylor series approximation (from Vol. II).

For well-graded crushed materials:

$$\begin{aligned} CV^2[k_c] = \frac{Var[k_c]}{\bar{k}_c^{\,2}} = 5.3018981 \Big[& h^2(0.08901252 - 0.0008850388h \\ & - 0.02901488 \log k_{sg})^2 CV^2[h] \\ & + (0.9640174 - 0.2561922 \log k_{sg} - \\ & - 0.012601\ h)^2\ CV^2[k_{sg}]\Big] \end{aligned} \tag{3.9}$$

where: $CV[i]$ = coefficient of variation of variable i

k_c = composite modulus of subgrade reaction, pci

h = layer thickness, in.

k_{sg} = modulus of subgrade reaction, pci

For natural sand and gravel:

$$CV^2[k_c] = 5.3018981 \left[h^2(0.08554373 - .0005149238h - 0.03050173 \log k_{sg})^2 CV^2[h] + (0.9829852 - 0.2617702 \log k_{sg} - 0.0132467h)^2 CV^2[k_{sg}]\right] \quad (3.10)$$

For stabilized layers:

$$CV^2[k_c] = 5.3018931\Big[\big[h^2(0.0544761 - 1.6947704\times10^{-3}h - 0.4409096\times10^{-2}\log k_{sg} -0.2465638\times10^{-8}E)^2 CV^2[h] + E^2(0.7254749\times10^{-6} - 0.3874568\times10^{-12}E - 0.4601653\times10^{-7}\log k_{sg} - 0.2465638\times10^{-8}h)^2 CV^2[E] + (0.4465112-0.1914846\times10^{-2}h - 0.1998463\times10^{-7}E)^2 CV^2[k_{sg}]\Big] \quad (3.11)$$

where E = modulus of elasticity of the stabilized layer, psi

Equations 3.5 and 3.8 are used to compute the average DF and its coefficient of variation, using (i) equation 3.3 and (ii) material characteristics.

These equation can easily be numerically transformed into different reliability-number of coverage - slab thickness relationships. Two possibilities of implementing the above formulation are discussed in the next section.

Reliability-Number of Coverages Relationship for Different Slab Thicknesses

With the design factor assumed normally distributed, the number of coverages corresponding to $DF(1 + k.CV[DF])$ can be computed, and the probability of $DF \leq DF(1 + kCV[DF])$ is taken from the normal distribution.

In order to do this, equations had to be developed for the DF-Number of Coverages curve (Figure 3.1a, since initial failure is assumed).

Since each curve is bilinear, two equations were derived for each. The first equation defines the curve below 5000 coverages and the second equation defines the rest of the curve. Since these equations were developed independent of the modulus of subgrade reaction, ranges had to be assumed for each curve. These ranges are:

Curve	Range of k Assigned to Curve
k = 25 - 200	25 - 250
300	250 - 350
400	350 - 450
500	450 - 500

This part of the design procedure yields the reliabilities for a given slab thickness in terms of the number of coverages it will take to produce that reliability.

Reliability-Slab Thickness Relationship for Different Number of Coverages

This part of the design procedure gives the reliability as a function of the slab thickness for a given number of coverages. This can be done by basically working the previous procedure in the reverse order. If the desired number of coverages is known, then the DF may also be calculated by using the equations described in the previous section. The probability is then taken from the normal distribution. The information obtained from this analysis can be very useful. If a certain reliability is desired for a given coverage level, then the necessary slab thickness can be found.

Study of Effects of the Variation of the Design Parameters

The closed form solution has the advantage to bring an insight of the effect of the design parameters. In eq. (3.8), the coefficient of variation of DF and hence the reliability of the design, is expressed as a function of the coefficient of variation of the design parameters. The effect of these parameters is enhanced if equation (3.8) is written as follows:

$$CV^2[DF] = \sum_i w_i \cdot CV^2[X_i] + \sum_i \xi_i \cdot \rho[X_i, X_k] \cdot CV[X_i] \cdot CV[X_k] \tag{3.12}$$

where: w_i, ξ_i - weighting factors ($w_i = \left(\frac{a_1 - a_2/\ell}{4\sigma h^2}\right)^2$ for the variable E, $w_i = 1$ for the variable MR, etc.)

The effect of each parameter is either increased or reduced by the corresponding weighting factor which comes from the response function. It should be noted that the weighting factor is dependent upon load and material characteristics and slab thickness. An evaluation of the weighting factors for the thirteen Aircraft Group Indices is presented in Appendix II, over the practical range of ℓ values. It is seen that the weighting factors of E and k (the modulus of elasticity of the concrete and the subgrade modulus of reaction) are quite small, resulting in a minor effect of their variability on the reliability of the design. The weighting factor of the slab thickness is the largest one, with the result of amplifying the effect of thickness variability on DF variability. However, it should be remembered that thickness variability is relatively small. It seems that the leading parameters controlling DF variability are thus the

modulus of rupture and load transfer variabilities. With the increase of number of wheels (from light to heavy-load design), the contribution of the variability of E and k increases.

Run Example

The new approach was used to develop reliability-number of coverages curves for different slab thicknesses (11, 12, 13, 14, 15 and 16 inch) for AGI 13. Reliability-slab thickness curves were then developed for different coverage levels (1000, 5000, 10,000, 20,000, 50,000, and 100,000 coverages). The average values of the variables and their coefficients of variation are given in Figure 3.2a. Output may be found in Figure 3.2b. Graphical results (plots of Reliability-Coverages-Slab Thickness relationships) may be found in Figure 3.3 and 3.4.

```
RUN EXAMPLE USAF AGI 13 INPUT DATA
***********************************

13  1       250.        0.1  4000000.        0.15
 3        12.       0.05   500000.       0.1
       11.        16.        1.
      800.         .1       .75          .1        .05        .02        .8
 4 1 1
  1000.   5000.  10000.  20000.  50000. 100000.
```

Figure 3.2a

Input Data for Run Example

USAF AGI-13

```
              MULTI-LAYERED SYSTEM ANALYSIS

SUBGRADE:
          MODULUS OF SUBGRADE REACTION =  250.00 PCI     CV =   .1000
    BASE:
          TYPE: STABILIZED
          HEIGHT = 12.00 INCHES        CV =   .0500
          MODULUS =  50000.00 PSI      CV =   .1000


COMPOSITE:
 LAYER  :
          MODULUS OF SUBGRADE REACTION =  905.07 PCI     CV =   .1098

                A MAXIMUM DESIGN K OF 500 PCI IS USED
```

Figure 3.2b Run Example Output

```
USAF AIRCRAFT GROUP INDEX: 1[illegible]
      A0           A1           A2          H2          B
  -517727.      265062.     425860.      1.000       1.000
NO.   THICK    SIGMA       DF       CVDF       FACT1     FACT2
 1     11.00  1182.20    .9023      .1401      .2291   -1.3127
         RELIA          DF                N
        .9987         .5231               1.
        .9974         .5484               1.
        .9953         .5737               2.
        .9918         .5990               4.
        .9861         .6243               7.
        .9772         .6495              12.
        .9641         .6748              22.
        .9452         .7001              40.
        .9192         .7254              71.
        .8849         .7516             127.
        .8413         .7759             228.
        .7881         .8012             408.
        .7257         .8265             730.
        .6554         .8517            1307.
        .5793         .8770            2339.
        .5000         .9023            4186.
        .4207         .9276            5896.
        .3446         .9528            7469.
        .2743         .9781            9463.
        .2119        1.0034           11989.
        .1587        1.0287           15189.
        .1151        1.0539           19248.
        .0808        1.0792           24381.
        .0548        1.1045           30888.
        .0359        1.1298           39133.
        .0228        1.1550           49579.
        .0139        1.1803           62813.
        .0082        1.2056           79579.
        .0047        1.2309          100823.
        .0026        1.2562          127733.
        .0013        1.2814          161828.
NO.   THICK    SIGMA       DF       CVDF       FACT1     FACT2
 2     12.00  1054.78   1.0113      .1401      .2301   -1.3097
         RELIA          DF                N
        .9987         .5864               3.
        .9974         .6147               6.
        .9953         .6430              11.
        .9918         .6715              21.
        .9861         .6997              39.
        .9772         .7280              76.
        .9641         .7563             145.
        .9452         .7847             279.
        .9192         .8130             534.
        .8849         .8413            1028.
        .8413         .8696            1974.
        .7881         .8980            3790.
        .7257         .9263            5827.
        .6554         .9546            7596.
        .5793         .9829            9902.
        .5000        1.0113           12908.
        .4207        1.0396           16827.
        .3446        1.0679           21936.
        .2743        1.0963           28597.
        .2119        1.1246           37279.
        .1587        1.1529           48598.
        .1151        1.1812           63353.
        .0808        1.2096           82588.
        .0548        1.2379          107666.
        .0359        1.2662          140352.
        .0228        1.2945          182966.
        .0139        1.3229          238519.
        .0082        1.3512          310937.
        .0047        1.3795          405343.
        .0026        1.4078          528413.
        .0013        1.4362          688851.
```

Figure 3.2b Run Example Output

NO.	THICK	SIGMA	UF	CVUF	FACT1	FACT2
3	13.00	945.76	1.1271	.1401	.2295	-1.3110

RELIA	UF	N
.9987	.6512	13.
.9974	.6824	27.
.9953	.7141	55.
.9918	.7456	113.
.9861	.7770	234.
.9772	[illegible]	483.
.9641	.8399	954.
.9452	.8714	2056.
.9192	.9029	4247.
.8849	.9343	6282.
.8413	.9658	[illegible]
.7881	.9972	11321.
.7257	1.0287	15153.
.6554	1.0617	20833.
.5793	1.0916	[illegible]
.5000	1.1231	[illegible]
.4207	1.1546	[illegible]
.3446	1.1861	66254.
.2743	1.2175	88582.
.2119	1.2490	119399.
.1587	1.2804	[illegible]
.1151	1.3119	215171.
.0808	1.3433	[illegible]
.0548	1.3748	387765.
.0359	1.4062	521585.
.0228	1.4377	698912.
.0139	1.4692	[illegible]
.0082	1.5006	1250331.
.0047	1.5321	1690567.
.0026	1.5636	2269873.
.0013	1.5950	3044617.

NO.	THICK	SIGMA	UF	CVUF	FACT1	FACT2
4	14.00	861.65	1.2335	.1401	.2278	-1.3165

RELIA	UF	N
.9987	.7177	65.
.9974	.7573	133.
.9953	.7878	265.
.9918	.8217	555.
.9861	.8564	1056.
.9772	.8911	[illegible]
.9641	.9258	[illegible]
.9452	.9605	8022.
.9192	.9951	11155.
.8849	1.0298	15357.
.8413	1.0645	21247.
.7881	1.0992	29396.
.7257	1.1339	40672.
.6554	1.1686	56277.
.5793	1.2033	77865.
.5000	1.2379	107717.
.4207	1.2726	149033.
.3446	1.3073	206196.
.2743	1.3420	285284.
.2119	1.3767	394707.
.1587	1.4114	546100.
.1151	1.4461	755561.
.0808	1.4807	[illegible]
.0548	1.5154	1446321.
.0359	1.5501	2001065.
.0228	1.5848	2768595.
.0139	1.6195	3830512.
.0082	1.6542	5299736.
.0047	1.6889	7332458.
.0026	1.7235	[illegible]
.0013	1.7582	[illegible]

NO.	THICK	SIGMA	UF	CVUF	FACT1	FACT2
5	15.00	786.66	1.3559	.1401	.2256	-1.3232

RELIA	UF	N
.9987	.7959	297.
.9974	.8316	[illegible]

Figure 3.2b (cont'd)

.5987	.[illegible]	1653.
.9938	.[illegible]	3206.
.9861	.9175	6457.
.9772	.9759	9272.
.9641	1.0115	[illegible]
.9452	1.0519	18884.
.9192	1.0855	[illegible]
.8849	1.1273	38469.
.8413	1.1665	54501.
.7881	1.2113	78354.
.7257	1.2415	[illegible]
.6554	1.2793	159594.
.5793	1.3175	227765.
.5000	1.3559	325068.
.4207	1.3939	463525.
.3446	1.4319	662109.
.2743	1.4700	[illegible]
.2119	1.5141	[illegible]
.1587	1.5446	[illegible]
.1151	1.5849	2786892.
.0808	1.6220	[illegible]
.0548	1.6611	5594067.
.0359	1.6960	[illegible]
.0228	1.7361	11195031.
.0139	1.7740	16260156.
.0082	1.8121	23211845.
.0047	1.8500	33127426.
.0026	1.8841	47278711.
.0013	1.9260	67475107.

NO.	THICK	SIGMA	DF	CUDF	FACT1	FACT2
6	16.00	722.05	1.4772	.1462	.22[illegible]0	-1.3310

DELTA	DF	N
.5987	.8[illegible]	1042.
.9974	.8978	3743.
.9953	.9368	6552.
.9918	.9812	9654.
.9861	1.0217	[illegible]
.9772	1.0681	20981.
.9641	1.1065	[illegible]
.9452	1.1459	85519.
.9192	1.1873	67056.
.8849	1.2297	98815.
.8413	1.2701	[illegible]
.7881	1.3115	218519.
.7257	1.3530	[illegible]
.6554	1.3966	465744.
.5793	1.4358	[illegible]
.5000	1.4772	[illegible]
.4207	1.5186	1489978.
.3446	1.5611	2195445.
.2743	1.6014	[illegible]
.2119	1.6428	4766596.
.1587	1.6841	[illegible]
.1151	1.7257	10388894.
.0808	1.7671	[illegible]
.0548	1.8085	22468792.
.0359	1.8455	33107218.
.0228	1.8913	48782489.
.0139	1.9327	[illegible]
.0082	1.9741	105913466.
.0047	2.0154	156060736.
.0026	2.0572	229051718.
.0013	2.0[illegible]	338628228.

M	DELTA	DF	N
11.00	.6896	.8801	1000.
12.00	.8[illegible]	.6001	1000.
13.00	.9680	.8801	1000.
14.00	.9991	.6801	1000.
15.00	.9567	.6801	1000.
16.00	.9991	.8801	1000.
11.00	.8[illegible]	.5100	5000.
12.00	.7627	.9100	5000.

Figure 3.2b (cont'd)

13.00	.9123	.9100	5000.
14.00	.9707	.9100	5000.
15.00	.9905	.9100	5000.
16.00	.9985	.9100	5000.
11.00	.2549	.9880	10000.
12.00	.5763	.9880	10000.
13.00	.8117	.9880	10000.
14.00	.9288	.9880	10000.
15.00	.9749	.9880	10000.
16.00	.9914	.9880	10000.
11.00	.1089	1.0581	20000.
12.00	.3706	1.0581	20000.
13.00	.6604	1.0581	20000.
14.00	.8502	1.0581	20000.
15.00	.9415	1.0581	20000.
16.00	.9785	1.0581	20000.
11.00	.0224	1.1559	50000.
12.00	.1535	1.1559	50000.
13.00	.4171	1.1559	50000.
14.00	.6818	1.1559	50000.
15.00	.8517	1.1559	50000.
16.00	.9396	1.1559	50000.
11.00	.0048	1.2300	100000.
12.00	.0613	1.2300	100000.
13.00	.2684	1.2300	100000.
14.00	.5183	1.2300	100000.
15.00	.7061	1.2300	100000.
16.00	.8037	1.2300	100000.

Figure 3.2b (cont'd)

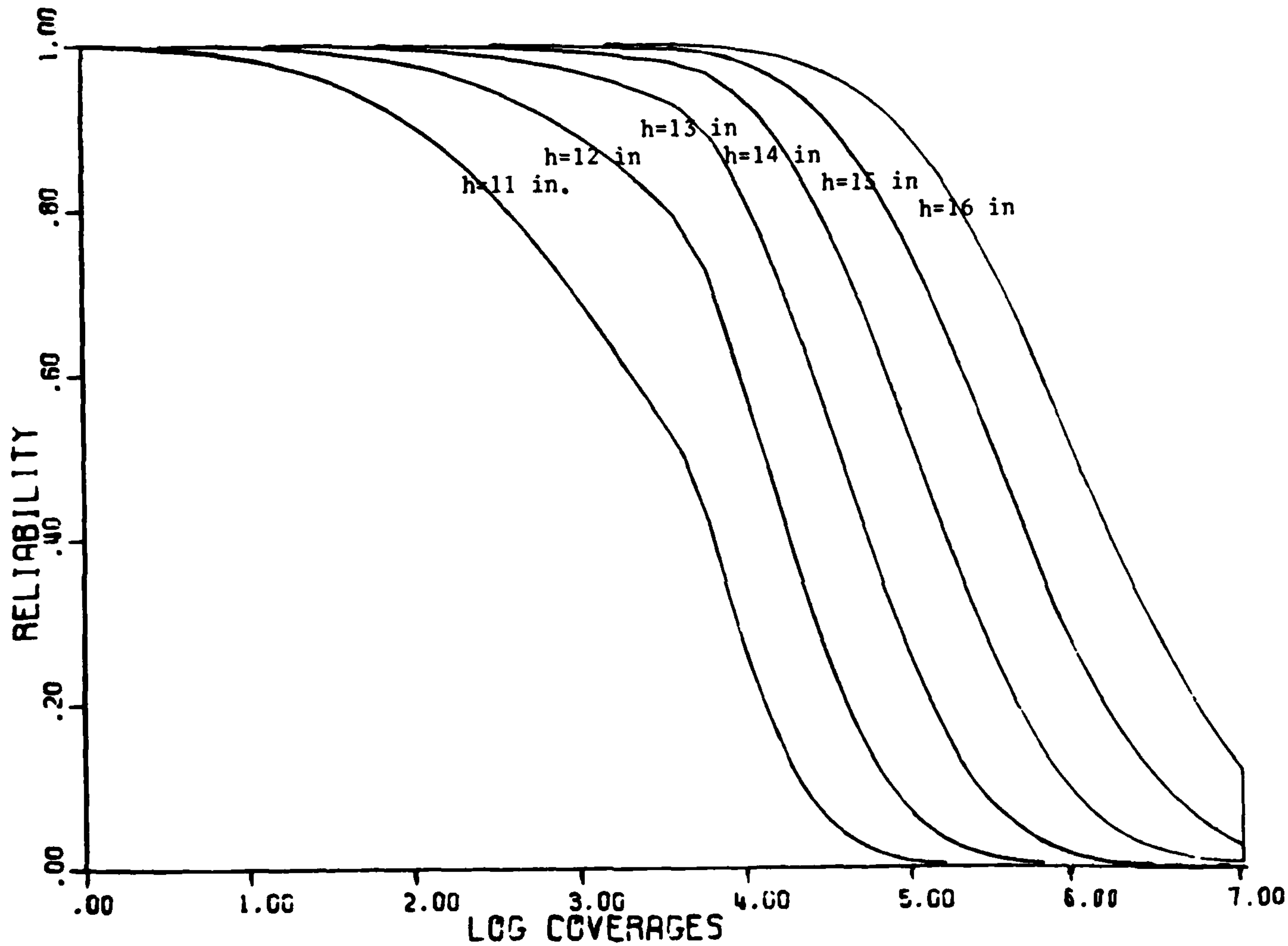

Figure 3.3

Reliability-Log Coverages Curves for
Different Slab Thicknesses

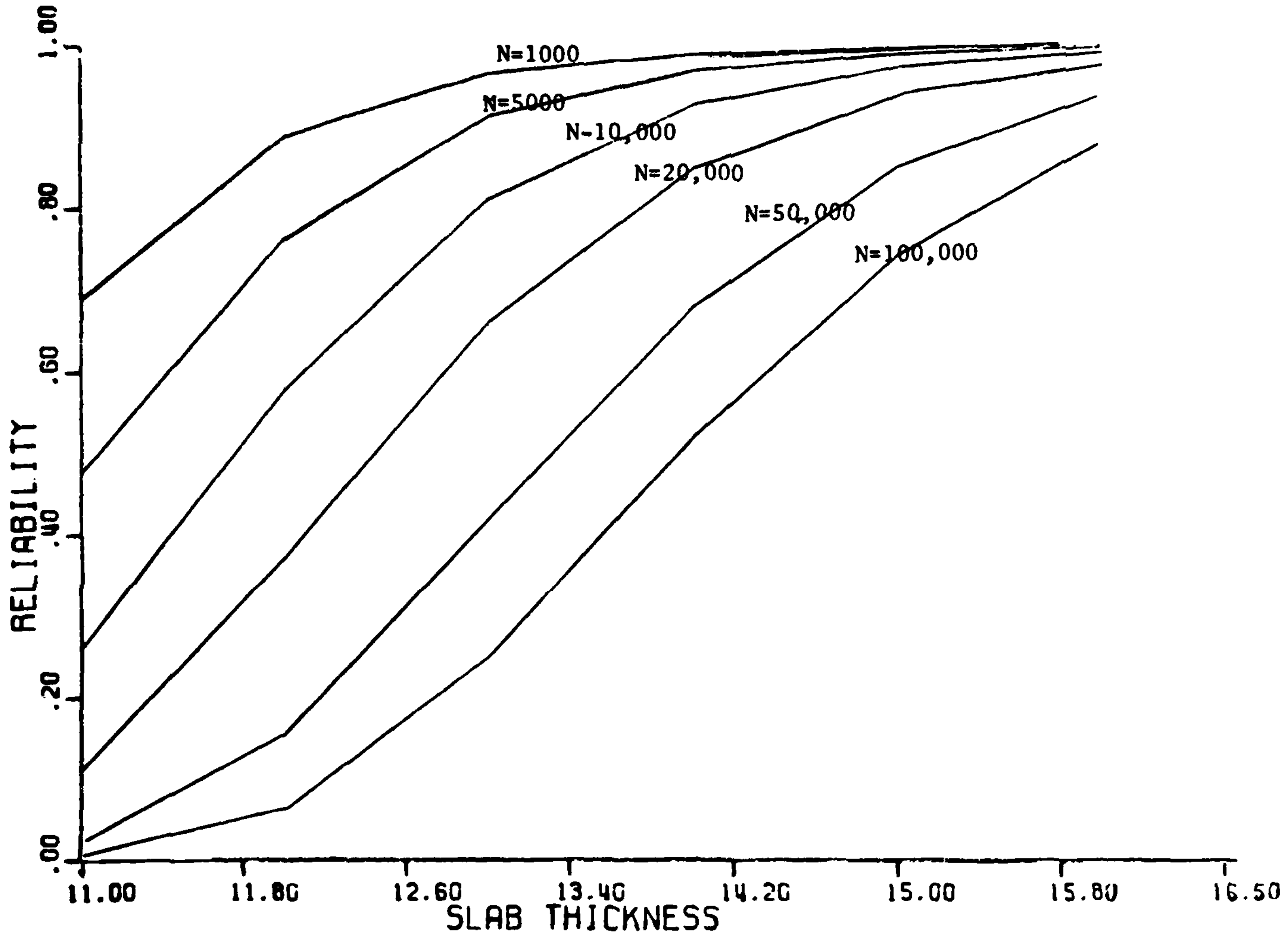

Figure 3.4

Reliability - Slab Thickness Curves
for Different Coverage Levels

Chapter 4

SUMMARY

The current U.S. Army and U.S. Air Force Design Manuals use the Westergaard free edge stress slab theory as the basis for their rigid pavement design methodologies. The design procedure has been expanded to provide a solution expressed in probabilistic and reliability terms. Further developments were required in the original procedure to make the analysis more practicable. Two major investigations were: (1) The use of the composite modulus of subgrade reaction to expand the procedure to solve problems having multiple subbase layers, and (2) The evaluation of a general equation form used to predict maximum tensile stresses at the bottom of the concrete slab for aircraft types designated by USAF AGI 1-15.

SPSS Multiple Regression runs were made to generate the composite modulus of subgrade reaction equations for three different material types currently used in military manuals. The correlations between the curve data and the equations that fit this data are very close to 1 (R^2 = 0.998, 0.997, and 0.997).

A regression equation was developed to predict the maximum tensile stress at the bottom of the concrete layer. Regression constants were then determined for each of the 1 USAF Aircraft Group Index controlling aircraft. The stresses predicted by this equation are extremely close to those predicted by the H-51 and well within the bounds of an acceptable margin of error.

The derivations above have been included in a computer program for the probabilistic/reliability analysis of rigid pavements. The approximate

closed-form probabilistic approach (Taylor Series) has been utilized. The computer program can be used:

(1) in the analysis in probabilistic/reliability terms for a given pavement system and loading aircraft. The means and coefficients of variation of the design parameters serve as the input. The computer program produces values of the number of coverages and their reliability levels.

(2) in the design of a rigid pavement for a given reliability. Here, the number of coverages is also an input so that the slab thickness may be determined for the given reliability and coverage level.

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5. Westergaard, H.M., "New Formulas for Stresses in Concrete Pavements of Airfields", Transactions, ASCE, Vol. 113, 1948, p. 425-439.
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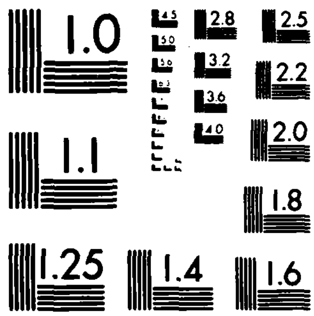
APPENDIX I

SPSS Outputs for the Composite Modulus Equations

AD-A138 212 DEVELOPMENT OF PROBABILISTIC RIGID PAVEMENT DESIGN METHODOLOGIES FOR MILI..(U) MARYLAND UNIV COLLEGE PARK DEPT OF CIVIL ENGINEERING M W WITCZAK ET AL. DEC 83 2/3

UNCLASSIFIED WES/TR/GL-83-18 F/G 13/13 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

COMPOSITE SUBGRADE MODULUS - II 09/23/82 PAGE 7

FILE NONAME (CREATION DATE = 09/23/82)

* M U L T I P L E R E G R E S S I O N * * * * * * * * * * * * * VARIABLE LIST 1
REGRESSION LIST 1

DEPENDENT VARIABLE.. AK1

VARIABLE(S) ENTERED ON STEP NUMBER 5.. H2

MULTIPLE R	.99919	ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
R SQUARE	.99838	REGRESSION	5.	7.27740	1.45548	9107.95893
ADJUSTED R SQUARE	.99827	RESIDUAL	74.	.01183	.00016	
STANDARD ERROR	.01264					

------------------ VARIABLES IN THE EQUATION ------------------

VARIABLE	B	BETA	STD ERROR B	F
AK2	.2219732+001	2.88249	.04751	2181.259
N	.8901252-001	1.35936	.00244	1325.502
AK4	-.2901888-001	-1.00888	.00078	1376.019
AK3	-.2949522+000	-1.49152	.01194	609.828
H2	-.4825194-003	-.17120	.00007	34.998
(CONSTANT)	-.1251102+001			

------------- VARIABLES NOT IN THE EQUATION -------------

VARIABLE	BETA IN	PARTIAL	TOLERANCE	F

MAXIMUM STEP REACHED

STATISTICS WHICH CANNOT BE COMPUTED ARE PRINTED AS ALL NINES.

Figure I-1 - SPSS Multiple Regression Output for the Composite Modulus of Subgrade Reaction of Well-Graded Crushed Materials

```
COMPOSITE SUBGRADE MODULUS - II                                                   09/29/82        PAGE    7
FILE    NONAME    (CREATION DATE = 09/29/82)
* * * * * * * * * * * * * * * * * * * *  M U L T I P L E    R E G R E S S I O N  * * * * * * * * * * * * * *   VARIABLE LIST  1
                                                                                                                REGRESSION LIST  1
DEPENDENT VARIABLE..    AK1
VARIABLE(S) ENTERED ON STEP NUMBER  5..    M2

MULTIPLE R            .99876                     ANALYSIS OF VARIANCE     DF     SUM OF SQUARES      MEAN SQUARE          F
R SQUARE              .99752                     REGRESSION               5.          7.18436          1.43687      5961.43008
ADJUSTED R SQUARE     .99734                     RESIDUAL                74.           .01788           .00024
STANDARD ERROR        .01553

------------------ VARIABLES IN THE EQUATION ------------------          -------------- VARIABLES NOT IN THE EQUATION --------------

VARIABLE            B             BETA      STD ERROR B        F          VARIABLE      BETA IN      PARTIAL    TOLERANCE        F

AK2           .2243807+001      2.95451        .05837      1503.659
M             .9558773-001      1.31824        .00300       811.654
AK8          -.3050177-001     -1.06697        .00096      1008.221
AK7          -.3013781+000     -1.53317        .01467       427.119
M2           -.2578619-003      -.10021        .00009         7.958
(CONSTANT)   -.1296098+001

MAXIMUM STEP REACHED
STATISTICS WHICH CANNOT BE COMPUTED ARE PRINTED AS ALL NINES.
```

Figure I-2 - SPSS Multiple Regression Output for the Composite Modulus Subgrade Reaction of Natural Sand and Gravel (PI<8)

COMPOSITE SUBGRADE MODULUS - GENERAL EQUATION 10/06/82 PAGE 8

FILE NONAME (CREATION DATE = 10/06/82)

• M U L T I P L E R E G R E S S I O N • • • • • • • • • • • • • • VARIABLE LIST 1
REGRESSION LIST 1

DEPENDENT VARIABLE.. A1

VARIABLE(S) ENTERED ON STEP NUMBER 7.. X3

MULTIPLE R	.99796
R SQUARE	.99593
ADJUSTED R SQUARE	.99549
STANDARD ERROR	.01241

ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
REGRESSION	7.	2.45060	.35009	2273.02487
RESIDUAL	65.	.01001	.00015	

---------------- VARIABLES IN THE EQUATION ----------------

VARIABLE	B	BETA	STD ERROR B	F
X1	.1000310[illegible]+001	1.11170	.03000	1111.451
X9	-.1195310-004	-.01890	.00300	.351
X7	-.1191078-002	-.08500	.00263	.205
X8	-.8667768-007	-.11704	.00700	1.444
X6	-.1976795-012	-.29515	.00060	75.416
X5	.7195853-006	1.02027	.00100	87.447
X3	.2955280-001	.52730	.00613	23.298
(CONSTANT)	-.1209181-001			

------------- VARIABLES NOT IN THE EQUATION -------------

VARIABLE	BETA IN	PARTIAL	TOLERANCE	F
X2	-.81477	-.17889	.00075	2.106
X4	-.31331	-.62178	.0160C	4C.232

• •

VARIABLE(S) ENTERED ON STEP NUMBER 8.. X4

MULTIPLE R	.99878
R SQUARE	.99750
ADJUSTED R SQUARE	.99719
STANDARD ERROR	.00980

ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
REGRESSION	8.	2.45486	.30691	3190.92620
RESIDUAL	64.	.00615	.00010	

---------------- VARIABLES IN THE EQUATION ----------------

VARIABLE	B	BETA	STD ERROR B	F
X1	.1028130+001	1.16300	.02419	1820.579
X9	-.2025438-008	-.03016	.00000	2.350
X7	-.8809024-002	-.16457	.00216	8.351
X8	-.8601453-007	-.11571	.00000	2.292
X6	-.1937293-012	-.28926	.00000	120.416
X5	.7258700-006	1.03410	.00000	77.872
X3	.5007610-001	.97200	.00624	76.278
X4	-.8873852-003	-.91331	.00013	40.232
(CONSTANT)	-.1578667+000			

------------- VARIABLES NOT IN THE EQUATION -------------

VARIABLE	BETA IN	PARTIAL	TOLERANCE	F
X2	-.52231	-.28610	.00075	5.610

F-LEVEL OR TOLERANCE-LEVEL INSUFFICIENT FOR FURTHER COMPUTATION

STATISTICS WHICH CANNOT BE COMPUTED ARE PRINTED AS ALL NINES.

Figure I-3 - SPSS Multiple Regression Output for the Composite Modulus of Subgrade Reaction of Stabilized Layers

APPENDIX II

Regression Constants for Free Edge Stress Equations

(AGI 1 to AGI 13)

Table II.1 - Summary of Stress and Weighting Factor Computations for AGI-1/C-123

h (in.)	ℓ (in.)	σ - (psi) H-51	EQ	Weighing Factors $W_{MR}=W_{\alpha}$	$W_{EL}=W_k$	W_h	$\xi_{E,MR}$	Error as % of H-51
6.0	26.47	1212.38	1212.56	1.0000	0.0351	2.0681	-0.3746	+0.01
7.0	29.72	968.94	968.38	1.0000	0.0300	2.1916	-0.3464	-0.06
8.0	32.85	791.94	792.98	1.0000	0.0265	2.2840	-0.3258	+0.13
9.0	35.88	663.21	662.59	1.0000	0.0238	2.3636	-0.3084	-0.09
10.0	38.83	563.20	562.93	1.0000	0.0218	2.4258	-0.2950	-0.05
11.0	41.71	484.50	484.90	1.0000	0.0202	2.4765	-0.2842	+0.08
12.0	44.52	422.35	422.55	1.0000	0.0188	2.5230	-0.2744	+0.05
13.0	47.27	372.17	371.89	1.0000	0.0177	2.5642	-0.2658	-0.07
14.0	49.98	330.12	330.18	1.0000	0.0167	2.5979	-0.2588	+0.02

a_o = - 70626 $\qquad$ R = 1.000

a_1 = 34374 $\qquad$ R^2 = 1.000

a_2 = 44166

k = 150 pci

Table II.2 - Summary of Stress and Weighting Factor Computations for AGI-2/F-4

h		σ - (psi)		Weighing Factors				Error as % of H-51
(in.)	(in.)	H-51	EQ	$W_{MR}=W_{\sigma}$	$W_{EL}=W_k$	W_h	$\varepsilon_{E,MR}$	
6.0	26.47	1676.00	1673.95	1.0000	0.0221	2.4165	-0.2970	-0.12
7.0	29.72	1311.20	1312.76	1.0000	0.0179	2.5565	-0.2674	+0.12
8.0	32.85	1056.24	1057.95	1.0000	0.0151	2.6611	-0.2458	+0.16
9.0	35.88	872.14	871.61	1.0000	0.0131	2.7450	-0.2288	-0.06
10.0	38.83	731.12	731.24	1.0000	0.0117	2.8090	-0.2160	+0.02
11.0	41.71	623.84	622.81	1.0000	0.0105	2.8646	-0.2050	-0.17
12.0	44.52	537.68	537.22	1.0000	0.0096	2.9084	-0.1964	-0.09
13.0	47.27	468.09	468.45	1.0000	0.0089	2.9453	-0.1892	+0.08
14.0	49.98	412.12	412.37	1.0000	0.0084	2.9784	-0.1828	+0.06

a_0 = 146

a_1 = 22439

a_2 = -354549

k = 150 pci

R = 1.000

R^2 = 1.000

Table II.3 - Summary of Stress and Weighting Factor Computations for AGI-3/F-111

h (in.)	ℓ (in.)	σ - (psi) H-51	σ - (psi) EQ	Weighing Factors $W_{MR}=W_{\alpha}$	Weighing Factors $W_{EL}=W_k$	Weighing Factors W_h	Weighing Factors $\varepsilon_{E,MR}$	Error as % of H-51
6.0	26.47	2122.98	2121.58	1.0000	0.0318	2.1465	-0.3566	-0.07
7.0	29.72	1685.46	1687.93	1.0000	0.0276	2.2560	-0.3320	+0.15
8.0	32.85	1378.84	1378.34	1.0000	0.0244	2.3461	-0.3122	-0.04
9.0	35.88	1150.03	1149.21	1.0000	0.0221	2.4165	-0.2970	-0.07
10.0	38.83	974.10	974.65	1.0000	0.0203	2.4728	-0.2850	+0.06
11.0	41.71	838.07	838.36	1.0000	0.0188	2.5230	-0.2744	+0.03
12.0	44.52	730.33	729.69	1.0000	0.0176	2.5670	-0.2652	-0.09
13.0	47.27	641.42	641.55	1.0000	0.0166	2.6027	-0.2578	+0.02
14.0	49.98	568.94	569.08	1.0000	0.0158	2.6358	-0.2510	+0.03

a_0 = -115728

a_1 = 57671

a_2 = 84016

k = 150 pci

R = 1.000

R^2 = 1.000

Table II.4 - Summary of Stress and Weighting Factor Computations for AGI-4/C-130

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_{\alpha}$	$W_{EL}=W_k$	W_h	$\varepsilon_{E,MR}$	
7.0	26.15	762.33	763.86	1.0000	0.0463	1.8344	-0.4304	+0.20
8.0	28.91	641.88	639.96	1.0000	0.0537	1.7028	-0.4634	-0.30
9.0	31.58	549.96	549.82	1.0000	0.0578	1.6353	-0.4808	-0.03
10.0	34.17	481.24	480.68	1.0000	0.0590	1.6162	-0.4858	-0.12
11.0	36.71	425.55	425.94	1.0000	0.0590	1.6162	-0.4858	+0.09
12.0	39.18	380.20	381.17	1.0000	0.0582	1.6292	-0.4824	+0.26
13.0	41.61	343.93	344.03	1.0000	0.0565	1.6569	-0.4752	+0.03
14.0	43.98	312.90	312.56	1.0000	0.0547	1.6864	-0.4676	-0.11
15.0	46.32	285.88	285.71	1.0000	0.0530	1.7145	-0.4604	-0.06

a_o = -332671 R = 1.000

a_1 = 94339 R^2 = 1.000

a_2 = 1626321

k = 250 pci

Table II.5 - Summary of Stress and Weighting Factor Computations for AGI-5/C-9

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_{\alpha}$	$W_{EL}=W_k$	W_h	$\varepsilon_{E,MR}$	
7.0	26.15	1240.10	1238.26	1.0000	0.0496	1.7748	-0.4452	-0.15
8.0	28.91	1033.77	1033.08	1.0000	0.0422	1.9149	-0.4108	-0.07
9.0	31.58	873.87	875.66	1.0000	0.0372	2.0209	-0.3856	+0.21
10.0	34.17	749.80	752.37	1.0000	0.0333	2.1098	-0.3650	+0.34
12.0	39.18	575.56	574.63	1.0000	0.0275	2.2569	-0.3318	-0.16
13.0	41.61	509.95	509.23	1.0000	0.0256	2.3113	-0.3198	-0.14
14.0	43.98	455.69	454.67	1.0000	0.0239	2.3608	-0.3090	-0.22
16.0	48.62	369.76	369.77	1.0000	0.0214	2.4380	-0.2924	+0.004
17.0	50.88	335.62	336.26	1.0000	0.0204	2.4690	-0.2858	+0.19

a_o = -127582 $\quad$ R = 1.000

a_1 = 56841 $\quad$ R^2 = 1.000

a_2 = 71556

k = 250pci

Table II.C - Summary of Stress and Weighting Factor Computations for AGI-6/T-43

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_{\alpha}$	$W_{EL}=W_k$	W_h	$\epsilon_{E,MR}$	
7.0	26.15	1247.20	1247.41	1.0000	0.0476	1.8109	-0.4362	+0.02
8.0	28.91	1041.67	1039.90	1.0000	0.0424	1.9108	-0.4118	-0.17
9.0	31.58	880.58	882.23	1.0000	0.0388	1.9861	-0.3938	+0.19
10.0	34.17	759.96	759.30	1.0000	0.0354	2.0604	-0.3764	-0.09
12.0	39.18	580.79	582.44	1.0000	0.0309	2.1677	-0.3518	+0.28
13.0	41.61	518.12	517.35	1.0000	0.0288	2.2219	-0.3396	-0.15
14.0	43.98	463.69	462.97	1.0000	0.0273	2.2632	-0.3304	-0.16
16.0	48.62	378.62	378.18	1.0000	0.0247	2.3351	-0.3146	-0.12
17.0	50.88	344.08	344.62	1.0000	0.0238	2.3627	-0.3086	+0.16

a_o = -183723   R = 1.000

a_1 = 69928 R^2 = 1.000

a_2 = 434391

k = 250 pci

Table II.7 - Summary of Stress and Weighting Factor Computations for AGI-7/B-727

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_{\alpha}$	$W_{EL}=W_{k}$	W_h	$\varepsilon_{E,MR}$	
8.0	26.58	1415.27	1414.53	1.0000	0.0517	1.7366	-0.4548	-0.05
9.0	29.03	1207.37	1209.18	1.0000	0.0480	1.8028	-0.4382	+0.15
11.0	33.75	920.14	919.40	1.0000	0.0415	1.9282	-0.4076	-0.08
13.0	38.25	728.09	726.88	1.0000	0.0368	2.0286	-0.3838	-0.17
15.0	42.58	589.67	591.66	1.0000	0.0336	2.1037	-0.3664	+0.34
17.0	46.77	493.45	492.57	1.0000	0.0304	2.1818	-0.3486	-0.18
19.0	50.84	417.62	417.52	1.0000	0.0282	2.2389	-0.3358	-0.02
20.0	52.84	386.65	386.62	1.0000	0.0272	2.2659	-0.3298	-0.01
21.0	54.81	359.03	359.17	1.0000	0.0263	2.2904	-0.3244	+0.04

a_0 = -348726   R = 1.000

a_1 = 121875 R^2 = 1.000

a_2 = 1049552

k = 350 pci

Table II.8 - Summary of Stress and Weighting Factor Computations for AGI-8/E-3

h (in.)	ℓ (in.)	σ - (psi) H-51	σ - (psi) EQ	Weighing Factors $W_{MR}=W_{\alpha}$	Weighing Factors $W_{EL}=W_k$	Weighing Factors W_h	Weighing Factors $\xi_{E,MR}$	Error as % of H-51
8.0	26.58	1006.29	1008.38	1.0000	0.0525	1.7224	-0.4584	+0.21
9.0	29.03	869.08	868.61	1.0000	0.0668	1.5001	-0.5168	-0.05
11.0	33.75	687.93	685.33	1.0000	0.0789	1.3393	-0.5618	-0.38
13.0	38.25	565.64	565.66	1.0000	0.0807	1.3179	-0.5680	+0.004
15.0	42.58	478.17	479.41	1.0000	0.0778	1.3526	-0.5580	+0.26
17.0	46.77	413.21	413.74	1.0000	0.0731	1.4132	-0.5408	+0.13
19.0	50.84	362.05	361.93	1.0000	0.0682	1.4796	-0.5224	-0.03
20.0	52.84	340.47	339.97	1.0000	0.0658	1.5141	-0.5130	-0.15
21.0	54.81	320.04	320.08	1.0000	0.0639	1.5423	-0.5054	+0.01

a_o = -823985　　　R = 1.000

a_1 = 221392　　　R^2 = 1.000

a_2 = 4314471

k = 350 pci

Table II.9 - Summary of Stress and Weighting Factor Computations for AGI-9/C-141

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_{\alpha}$	$W_{EL}=W_k$	W_h	$\epsilon_{E,MR}$	
8.0	26.58	1027.58	1029.53	1.0000	0.0783	1.3463	-0.5598	+0.19
9.0	29.03	902.06	900.45	1.0000	0.0866	1.2479	-0.5886	-0.18
11.0	33.75	722.64	721.84	1.0000	0.0897	1.2133	-0.5990	-0.11
13.0	38.25	598.93	599.54	1.0000	0.0854	1.2614	-0.5846	+0.10
15.0	42.58	509.12	509.24	1.0000	0.0789	1.3393	-0.5618	+0.02
17.0	46.77	439.47	439.61	1.0000	0.0726	1.4197	-0.5390	+0.03
19.0	50.84	384.38	384.35	1.0000	0.0669	1.4987	-0.5172	-0.01
20.0	52.84	360.99	360.87	1.0000	0.0643	1.5364	-0.5070	-0.03
21.0	54.81	339.60	339.58	1.0000	0.0620	1.5708	-0.4978	-0.005

a_o = -802693 $\qquad$ R = 1.000

a_1 = 220139 $\qquad$ R^2 = 1.000

a_2 = 3893752

k = 350 pci

Table II.10 Summary of Stress and Weighting Factor Computations for AGI-10/C-5A

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_{\alpha}$	$W_{EL}=W_k$	W_h	$\xi_{E,MR}$	
8.0	26.58	838.15	842.22					+0.49
9.0	29.03	719.03	715.91					-0.43
11.0	33.75	556.58	554.84	Weighting factors not given due to the non-symmetric nature of the C-5A gear configuration				-0.31
13.0	38.25	454.00	453.11					-0.20
15.0	42.58	379.20	381.48					+0.60
17.0	46.77	327.63	327.78					+0.05
19.0	50.84	286.07	285.88					-0.07
20.0	52.84	268.25	268.23					-0.01
21.0	54.81	252.59	252.29					-0.12

a_o = -653058

a_1 = 174658

a_2 = 3563166

k = 350 pci

Table II.11 - Summary of Stress and Weighting Factor Computations for AGI-11/KC-10

h (in.)	ℓ (in.)	σ - (psi) H-51	σ - (psi) EQ	Weighing Factors $W_{MR}=W_{\alpha}$	$W_{EL}=W_k$	W_h	$\epsilon_{E,MR}$	Error as % of H-51
8.0	26.58	1089.90	1094.80	1.0000	0.0344	2.0846	-0.3708	+0.45
9.0	29.03	935.34	930.83	1.0000	0.0517	1.7374	-0.4546	-0.48
11.0	33.75	726.64	725.47	1.0000	0.0727	1.4182	-0.5394	-0.16
13.0	38.25	596.92	596.81	1.0000	0.0794	1.3331	-0.5636	-0.02
15.0	42.58	505.14	505.94	1.0000	0.0794	1.3338	-0.5634	+0.16
17.0	46.77	436.60	437.35	1.0000	0.0763	1.3722	-0.5524	+0.17
19.0	50.84	383.50	383.43	1.0000	0.0720	1.4275	-0.5368	-0.02
20.0	52.84	360.37	360.58	1.0000	0.0701	1.4542	-0.5294	+0.06
21.0	54.81	340.47	339.88	1.0000	0.0678	1.4862	-0.5206	-0.17

a_o = -968715   R = 1.000

a_1 = 254788 R^2 = 1.000

a_2 = 5396732

k = 350 pci

Table II.12 - Summary of Stress and Weighting Factor Computations for AGI-12/E-4

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_{\alpha}$	$W_{EL}=W_{k}$	W_{h}	$\xi_{E,MR}$	
9.0	26.55	833.52	839.32	1.0000	0.0377	2.0090	-0.3884	+0.70
11.0	30.87	650.71	646.26	1.0000	0.0683	1.4789	-0.5226	-0.68
13.0	34.99	536.25	531.93	1.0000	0.0825	1.2987	-0.5736	-0.81
15.0	38.95	452.55	453.09	1.0000	0.0875	1.2379	-0.5916	+0.12
17.0	42.78	392.51	394.00	1.0000	0.0866	1.2479	-0.5886	+0.38
19.0	46.51	345.76	347.59	1.0000	0.0834	1.2850	-0.5776	+0.53
21.0	50.13	308.94	309.80	1.0000	0.0788	1.3407	-0.5614	+0.28
23.0	53.67	278.86	278.49	1.0000	0.0739	1.4026	-0.5438	-0.13
24.0	55.41	265.82	264.74	1.0000	0.0715	1.4347	-0.5348	-0.41

a_o = -1018918 $\qquad$ R = 1.000

a_1 = 266253 $\qquad$ R^2 = 1.000

a_2 = 5677764

k = 500 pci

Table II.13 - Summary of Stress and Weighting Factor Computations for AGI-13/B-52

h (in.)	ℓ (in.)	σ - (psi) H-51	EQ	Weighing Factors $W_{MR}=W_\alpha$	$W_{EL}=W_k$	W_h	$\xi_{E,MR}$	Error as % of H-51
9.0	26.55	1537.84	1542.53	1.0000	0.0476	1.8109	-0.4362	+0.30
11.0	30.87	1185.48	1182.31	1.0000	0.0522	1.7279	-0.4570	-0.27
13.0	34.99	953.73	949.83	1.0000	0.0522	1.7279	-0.4570	-0.41
15.0	38.95	786.10	786.64	1.0000	0.0510	1.7493	-0.4516	+0.07
17.0	42.78	665.87	665.89	1.0000	0.0485	1.7940	-0.4404	+0.003
19.0	46.51	570.58	573.27	1.0000	0.0464	1.8328	-0.4308	+0.47
21.0	50.13	499.64	500.01	1.0000	0.0436	1.8860	-0.4178	+0.07
23.0	53.67	441.16	440.95	1.0000	0.0413	1.9332	-0.4064	-0.05
25.0	57.14	393.27	392.48	1.0000	0.0392	1.9777	-0.3958	-0.20

a_o = -917727

a_1 = 269062

a_2 = 4258868

k = 500 pci

R = 1.000
R^2 = 1.000

Table II - 14 - Summary of Stress and Weighting Factor Computations

For AGI - 14/OV-1

h (in.)	ℓ (in.)	σ - (psi)		Weighting Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_a$	$W_{EL}=W_k$	W_h	$\varepsilon_{E,MR}$	
6.0	26.47	561.83	563.00	1.0000	0.0147	2.6758	-0.2420	+0.21
7.0	29.72	437.56	436.77	1.0000	0.0130	2.7510	-0.2276	-0.18
8.0	32.85	350.14	351.02	1.0000	0.0117	2.8060	-0.2166	+0.25
9.0	35.88	287.58	286.81	1.0000	0.0108	2.8514	-0.2076	-0.27
10.0	38.83	239.66	239.94	1.0000	0.0101	2.8839	-0.2012	+0.12
11.0	41.71	203.41	204.00	1.0000	0.0095	2.9135	-0.1954	+0.29
12.0	44.52	175.35	175.76	1.0000	0.0090	2.9412	-0.1900	+0.24
13.0	47.27	153.13	153.13	1.0000	0.0086	2.9670	-0.1850	+0.02
14.0	49.98	135.17	134.78	1.0000	0.0081	2.9908	-0.1804	-0.29

a_o = -9544. R = 0.999

a_1 = 9268. R^2 = 1.000

a_2 = 14558.

k = 150 psi

Table II - 15 - Summary of Stress and Weighting Factor Computations

For AGI - 15/C-54

h (in.)	ℓ (in.)	σ - (psi)		Weighting Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_a$	$W_{EL}=W_k$	W_h	$\epsilon_{E,MR}$	
7.0	26.15	635.12	635.23	1.0000	0.0301	2.1880	-0.3472	+0.02
8.0	28.91	520.93	520.87	1.0000	0.0283	2.2362	-0.3364	-0.01
9.0	31.58	436.59	436.38	1.0000	0.0266	2.2813	-0.3264	-0.05
10.0	34.17	372.55	371.88	1.0000	0.0252	2.3232	-0.3172	-0.18
11.0	36.71	320.77	321.48	1.0000	0.0241	2.3553	-0.3102	+0.22
12.0	39.18	280.57	281.12	1.0000	0.0230	2.3886	-0.3030	+0.20
13.0	41.61	248.55	248.31	1.0000	0.0218	2.4239	-0.2954	-0.10
14.0	43.98	221.79	221.15	1.0000	0.0209	2.4549	-0.2888	-0.29
15.0	46.32	198.15	198.46	1.0000	0.0202	2.4756	-0.2844	+0.16

a_o = 76186. R = 1.000

a_1 = 30235. R^2 = 1.000

a_2 = 225665.

k = 250 psi

APPENDIX III

USER'S GUIDE

USER'S GUIDE

Card 1 (I2, I3, 4F10.4)

1-2	IG	- USAF Aircraft Group Index (=0 for user defined aircraft)
3-5	NOLBP	- Number of base (subbase) layers (excludes subgrade, maximum of 2)
6-15	XK	- Modulus of subgrade reaction, pci
16-25	CVK	- Coefficient of variation of XK
26-35	ETEMP	- Modulus of elasticity of the pavement, psi
36-45	XMUTEM	- Poisson's ratio of the pavement

Card 2 (I2, 4F10.4) Repeat NOLBP times *See notes

1-2	MATOPT	- Material of the base (subbase) layer
		= 1 for well-graded crushed materials
		= 2 for natural sands and gravels (PI 8)
		= 3 for stabilized materials
3-12	HB	- Thickness of base (subbase) layer, in.
13-22	CVHB	- Coefficient of variation of HB
23-32	EB	- Modulus of elasticity of base (subbase) layer, psi (only required when MATOPT = 3)
33-42	CVEB	- Coefficient of variation of EB (only required when MATOPT = 3)

Card 3 (10A6) Required if IG = 0 *See notes

1-60		- Comments (Only input if aircraft is user defined)

Card 4 (4F10.0) Required if IG = 0

1-10	G	- Load on gear, lb.
11-20	P	- Tire inflation pressure, psi
21-30	A	- Contact area of one tire, sq. in.
31-40	XNOG	- Number of γ (GAMMA)

Card 5 (6F10.0) Required if IG = 0

1-10	XLA	- Gear spacing parameters a, b, c, d, in.
11-20	XLB	
21-30	XLC	
31-40	XLD	
41-50	XNOD	- Number of Δ's (DELTA)
51-60	NXOSG	-Number of (σ,G)'s (ASIG, AG)

Card 6 (5F10.0) Required if IG = 0

1-10	XNA	Gear spacing parameters n_a, n_b, n_c, n_d
11-20	XNB	
21-30	XNC	
31-40	XND	
41-50	PHIE	- Ratio of length to width of the tire print (optional)

Card 7 (5F10.0) Required if IG = 0

1-10	B	- Number of tire approximation points
11-20	XOP3	- Print option. Enter 1.0 to print subtotals for each wheel
21-30	BIGX	- Location of stress calculation point relative to gear, in.
31-40	BIGY	- Location of stress calculation point relative to gear, in.
41-50	XOP6	- Coordinate option. Enter 1.0 when using optional H-51 input method

Card 8 (10F6.0) Required if IG=0

1-6	GAMMA(1)	- Position of gear (angular), deg.
7-12	GAMMA(2)	
⋮	⋮	
	GAMMA(XNOG)	

Card 9 (10F6.0) Required if IG = 0

1-6 DELTA(1)

7-12 DELTA(2)

. .

. .

. .

DELTA(XNOD)

Card 10 (8F10.0) Repeat XNOSG times, Required if IG = 0

1-10	ASIG(I)	Input Pairs
11-20	AG(I)	

Card 11 (3F10.2)

1-10 HMIN - Minimum slab thickness to be evaluated, in.

11-20 HMAX - Maximum slab thickness to be evaluated, in.

21-30 HINCR - Increment by which the slab thickness is increased from HMIN to HMAX, in.

Card 12 (7F10.0)

1-10 AMR - Modulus of rupture of concrete, psi

11-20 CUMR - Coefficient of variation of AMR

21-30 ALPHA - Load transfer coefficient

31-40 CVALP - Coefficient of variation of ALPHA

41-50 CVE - Coefficient of variation of ETEMP

51-60 CVH - Coefficient of variation of slab thickness

61-70 ROEMR - Correlation coefficient between AMR and ETEMP

Card 13 (3I2)

1-2 NON - Number of N's (coverage levels) to be input

2-3 JOPPLI - Plot option. Enter 1 to plot Reliability vs log coverages

3-4 JOPPLZ - Plot option. Enter 1 to plot Reliability vs. Slab thickness

Card 14 (10F8.0)

1-8 BN(1) - Number of coverages

9-16 BN(2)

. .
. .
. .

BN(NON)

Notes

1. Card 2 should be omitted if no composite modulus of subgrade reaction is to be calculated.
2. If NOLBP = 2, the data for the layer directly above the subgrade should be input first.
3. In the output, Layer 2 will always be labeled the "BASE", and Layer 3 will always be labeled the "SUBBASE".
4. Cards 4-11 should only be included if the aircraft is user defined (i.e IG=0)
5. The parameters listed on cards 3-10 are defined in the same manner as in the H-51 program. Any additional information concerning these parameters may be found in the H-51 user's manual(4).

APPENDIX IV

PROGRAM LISTING

```
WZAN*NEW(1).NEST(3)
@FOR,IS MAIN
C                       N51 - MAIN PROGRAM
C
C      N51 COMPUTES BENDING STRESS IN CONCRETE
C
C
      COMMON /ABLK/ A,AP(10),AW(10),AK(10),ASIG(10),AG(10)
      COMMON /BBLK/ BIGX,BIGY,B
      COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)
      COMMON /EBLK/ E,ER(10)
      COMMON /FBLK/ FG(10)
      COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)
      COMMON /HBLK/ H(10)
      COMMON /IBLK/ ICR(10),ILM,IPT,IT,ILD,ISG,INOG    ,KBP
      COMMON /NBLK/ NOG,NOM,NPT,NX,NY,NOD,NOSG
      COMMON /PBLK/ P,PI,PVMTST(10),PHIE
      COMMON /RBLK/ RPD,RZERO
      COMMON /SBLK/ SIGMA(10),S,SR
      COMMON /TBLK/ T,TR
      COMMON /WBLK/ WR,WR2
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOM,XNOD,XNOSG,XNT,XOP1,
     2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),
     3              XZERO,XZEROP
      COMMON /YBLK/ YMN(10),WP(100,10) ,YPZ(4097),YZERO,YZEROP
      COMMON /N51AT/ KERN(5)
      COMMON /BLK1/ NW,NZ,CV(700)
      COMMON /UBLK/ UNUM(10,10),UMAX(10)
      COMMON /MPJ/  AO,A1,A2,ALPHA,AMR,BN(10),CVDDF(10),CVK,IG,NOM,
     1              JOPPL1,JOPPL2
      DIMENSION AA(13,3)
      DATA AA / -70626.,146.,-115728.,-332671.,-127362.,-187723.,
     A-348726.,-823985.,-802693.,-653058.,-968715.,-1018918.,-917727.,
     B-9544.,-76186.,34374.,22459.,57671.,94339.,56847.,69928.,121875.,
     C221392.,220139.,174658.,254788.,266253.,269062.,9268.,30235.,
     D44166.,-354549.,84016.,1626321.,71556.,434391.,1049552.,4314471.,
     E3293752.,3563166.,5396732.,5677764.,4258868.,-14558.,225665./
      DATA KPROC/3HN51/,KNAME/6HN51A/
   10 CONTINUE
  150 KERN(1) = KPROC
      KERN(2) = KNAME
      KERN(3) = 1
      KERN(4) = 0
      KERN(5) = 0
  200 FORMAT(I2,I3,4F10.4)
      READ(5,200,END=9999)IG,NOLBP,XK,CVK,ETEMP,XMUTEM
      IF(ETEMP.GT.0.)E=ETEMP
      IF(ETEMP.GT.0.)GO TO 400
      E=4000000.
  400 IF(XMUTEM.GT.0.)XMU=XMUTEM
      IF(XMUTEM.GT.0.)GO TO 500
      XMU=0.15
  500 CONTINUE
      IF(NOLBP.EQ.0)GO TO 1000
      CALL KCOMP
 1000 CONTINUE
 1010 IF(IG)2000,2000,2010
 2000 CALL PRSPRD
 2010 READ(5,210)HMIN,HMAX,HINCR
  210 FORMAT(3F10.2)
      H(1)=HMIN
      NOM=1
  220 IF(H(NOM).GE.HMAX)GO TO 300
      NOM=NOM+1
      H(NOM)=H(NOM-1)+HINCR
      GO TO 220
  300 CONTINUE
      IF(IG)2100,2100,2110
 2110 AO=AA(IG,1)
      A1=AA(IG,2)
      A2=AA(IG,3)
      GO TO 3000
 2100 DO 100 ILM=1,NOM
      CALL GEOM
      CALL OUTLNE
```

```
      IF(XOP6.GT.0.5)  GO TO  110
      UMAX(ILH)=UNUM(1,1)
      DO 101 IUZ=1,NOD
      DO 101 IJA=1,NOSG
      IF(UMAX(ILH).GT.UNUM(IUZ,IJA))  GO TO  101
      UMAX(ILH)=UNUM(IUZ,IJA)
  101 CONTINUE
      GO TO 100
  110 UMAX(ILH)=PVMTST(1)
  100 CONTINUE
      A0=0.
      A1=0.
      A2=0.
 3000 CALL RELIAB
      CALL STORPL
      GO TO 10
 9999 STOP
      END
C
C
@FOR,IS RELIAB
      SUBROUTINE RELIAB
C
C
      COMMON /ABLK/ A,AP(10),AH(10),AK(10),ASIG(10),AG(10)
      COMMON /BBLK/ BIGX,BIGY,B
      COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)
      COMMON /EBLK/ E,ER(100)
      COMMON /FBLK/ FG(10)
      COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)
      COMMON /HBLK/ H(10)
      COMMON /IBLK/ ICH(10),ILH,IPT,IT,ILD,ISG,INOG    ,KBP
      COMMON /NBLK/ NOG,NOH,NPT,NX,NY,NOD,NOSG
      COMMON /PBLK/ P,PI,PVMTST(10),PHIE
      COMMON /RBLK/ RPD,RZERO
      COMMON /SBLK/ SIGMA(10),S,SR
      COMMON /TBLK/ T,TR
      COMMON /WBLK/ WR,WR2
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOH,XNOD,XNOSG,XNT,XOP1,
     2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),
     3              XZERO,XZEROP
      COMMON /YBLK/ YMN(10),WP(100,10) ,YPZ(4097),YZERO,YZEROP
      COMMON /HS1A1/ KERN(5)
      COMMON /BLK1/ NW,NZ,CV(700)
      COMMON /UBLK/ UNUM(10,10),UMAX(10)
      COMMON /MPJ/  A0,A1,A2,ALPHA,AMR,BN(10),CVDDF(10),CVK,IG,NON,
     1              JOPPL1,JOPPL2
      DIMENSION A(3,3),AA(3,4),BB(2,3),DDF(33),PR(33),AN(33),B(3),JC(3),
     &          AL(10),V(3)
      DATA KPROC/3HHS1/,KNAME/4HHS1A/
      R=1.
      R2=1.
      DO 1 I=1,3
      B(I)=0.
      DO 1 J=1,3
      A(I,J)=0.
    1 CONTINUE
      READ(5,1000)AMR,CVMR,ALPHA,CVALP,CVE,CVH,ROEMR
      READ(5,70)NON,JOPPL1,JOPPL2
   70 FORMAT(3I2)
 1000 FORMAT(7F10.0)
      WRITE(6,1001)
 1001 FORMAT(1H1)
      IF(IG.EQ.0)GO TO 2000
      DO 2001 ILH=1,NOH
      AX=E*H(ILH)**3./11.73/XK
      AL(ILH)=AX**0.25
      UMAX(ILH)=A0+A1*ALOG(AL(ILH))+A2/AL(ILH)
 2001 CONTINUE
      GO TO 210
 2000 DO 10 ILH=1,NOH
      AX=E*H(ILH)**3./11.73/XK
      AL(ILH)=AX**0.25
      X1=ALOG(AL(ILH))
      X2=1./AL(ILH)
```

```
      UMAX(ILH)=0.0006*P*SQRT(AX)*UMAX(ILH)
      B(1)=B(1)+UMAX(ILH)
      A(1,1)=A(1,1)+1.
      A(1,2)=A(1,2)+X1
      A(1,3)=A(1,3)+X2
      B(2)=B(2)+X1*UMAX(ILH)
      A(2,2)=A(2,2)+X1*X1
      A(2,3)=A(2,3)+X1*X2
      B(3)=B(3)+X2*UMAX(ILH)
      A(3,3)=A(3,3)+X2*X2
      SYY=SYY+UMAX(ILH)*UMAX(ILH)
   10 CONTINUE
      A(2,1)=A(1,2)
      A(3,1)=A(1,3)
      A(3,2)=A(2,3)
      DO 20 I=1,3
      AA(I,4)=B(I)
      DO 20 J=1,3
   20 AA(I,J)=A(I,J)
      YAV=B(1)/A(1,1)
      V(1)=4.
      CALL GJR(AA,4,3,3,4,$100,JC,V)
      A0=AA(1,4)
      A1=AA(2,4)
      A2=AA(3,4)
      GO TO 150
  100 WRITE(6,1015)
      DO 130 I=1,2
      BB(I,3)=B(I)
      DO 130 J=1,2
  130 BB(I,J)=A(I,J)
      V(1)=4.
      CALL GJR(BB,3,2,2,3,$999,JC,V)
      A0=BB(1,3)
      A1=BB(2,3)
      A2=0.
  150 CONTINUE
      Y2EST=A(1,1)*A0*A0+A(2,2)*A1*A1+A(3,3)*A2*A2+2.*A0*A1*A(1,2)+
     &          2.*A0*A2*A(1,3)+2.*A1*A2*A(2,3)
      YAVN=YAV*A(1,1)
      Y2AVN=YAVN*YAV
      R2=(Y2EST-Y2AVN)/(SYY-Y2AVN)
      R=SQRT(R2)
      IF(IG)200,200,210
  210 WRITE(6,211)IG
  211 FORMAT(/,5X,'USAF AIRCRAFT GROUP INDEX: ',I2,/)
  200 WRITE(6,1010)  A0,A1,A2,R2,R
 1010 FORMAT('          A0          A1          A2        R2        R '/
     &           3F12.0,2F10.3)
      IF(JOPPL1.LT.1)GO TO 71
      CALL PLOTUZ(AN,PR,-1)
   71 CONTINUE
 2020 DO 50 I=1,NOM
      WRITE(6,1011)
      SIG=UMAX(I)/H(I)/H(I)
      FACT1=(A1-A2/AL(I))/4./UMAX(I)
      FACT2=-2.+3.*FACT1
      DF=AMR/ALPHA/SIG
      CVDF=CVMR*CVMR+CVALP*CVALP+FACT1*FACT1*(CVE*CVE+CVK*CVK)+
     &        FACT2*FACT2*CVH*CVH-2.*FACT1*RDEMR*CVE*CVMR
      CVDF=SQRT(CVDF)
      CVDDF(I)=CVDF
      WRITE(6,1012) I,H(I),SIG,DF,CVDF,FACT1,FACT2
 1011 FORMAT(/,1X,'NO.',3X,'THICK',4X,'SIGMA',5X,'DF',5X,'CVDF',5X,
     1'FACT1',4X,'FACT2')
 1012 FORMAT(I4,2F9.2,4F9.4)
      XX=3.
      WRITE(6,1013)
      DO 40 J=1,16
      ZX=EXP(-XX*XX/2.)/2.5066283
      TX=1./(1.+0.2316419*XX)
      PX=1.-ZX*TX*(0.31938153+TX*(-0.356563782+
     &             TX*(1.781477937+TX*(-1.821255978+TX*1.330274429))))
      DDF(J)=DF*(1.-CVDF*XX)
      PR(J) =PX
      DDF(32-J)=DF*(1.+CVDF*XX)
```

```
      PR(32-J) =1.-PX
      XX=XX-0.2
      II=J
      DO 39 K=1,2
      IF(XK.LT.250.)  GO TO 31
      IF(XK.LT.350.)  GO TO 33
      IF(XK.LT.450.)  GO TO 35
      IF(DDF(II).GT.0.91)  GO TO 37
      AN(II)=10.**((DDF(II)-0.5401)/0.1)
      GO TO 38
   37 AN(II)=10.**(DDF(II)/C.246)
      GO TO 38
   35 IF(DDF(II).GT.1.09)  GO TO 36
      AN(II)=10.**((DDF(II)-0.6415)/0.1186)
      GO TO 38
   36 AN(II)=10.**((DDF(II)-0.0565)/0.2767)
      GO TO 38
   33 IF(DDF(II).GT.1.21)  GO TO 34
      AN(II)=10.**((DDF(II)-0.7029)/0.1371)
      GO TO 38
   34 AN(II)=10.**((DDF(II)-0.0445)/0.3151)
      GO TO 38
   31 IF(DDF(II).GT.1.3) GO TO 32
      AN(II)=10.**((DDF(II)-0.7655)/0.1445)
      GO TO 38
   32 AN(II)=10.**((DDF(II)-0.049)/0.3382)
   38 II=32-J
   39 CONTINUE
   40 CONTINUE
      WRITE(6,1014)  ((PR(K),DDF(K),AN(K)),K=1,31)
      DO 42 J=1,31
      AN(J)=ALOG10(AN(J))
      IF(AN(J).LT.0.0001)  AN(J)=0.0001
      IF(AN(J).GT.6.999)  AN(J)=6.999
   42 CONTINUE
      IF(JOPPL1.LT.1)GO TO 72
      CALL PLOTUZ(AN,PR,I)
   72 CONTINUE
   50 CONTINUE
      GO TO 998
  999 WRITE(6,1015)
 1013 FORMAT('            RELIA        OF                N')
 1014 FORMAT(2F12.4,F15.0)
 1015 FORMAT('    ILL MATRIX !!!!!!!    ')
      STOP
  998 RETURN
      END
C
C
@FOR,IS FINISH
                   SUBROUTINE FINISH
C
C
      COMMON /ABLK/ A,AP(10),AN(10),AK(10),ASIG(10),AG(10)
      COMMON /BBLK/ BIGX,BIGY,B
      COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)
      COMMON /EBLK/ E,ER(100)
      COMMON /FBLK/ FG(10)
      COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)
      COMMON /HBLK/ H(10)
      COMMON /IBLK/ ICH(10),ILH,IPT,IT,ILD,ISG,INOG,  KBP
      COMMON /NBLK/ NOG,NOH,NPT,NX,NY,NOD,NOSG
      COMMON /PBLK/ P,PI,PVMTST(10),PHIE
      COMMON /RBLK/ RPD,RZERO
      COMMON /SBLK/ SIGMA(10),S,SR
      COMMON /TBLK/ T,TR
      COMMON /WBLK/ WR,WR2
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOH,XNOD,XNOSG,XNT,XOP1,
     2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),
     3              XZERO,XZEROP
      COMMON /YBLK/ YMN(10),WP(100,10) ,YPZ(4097),YZERO,YZEROP
      COMMON /HSTAT/ KERN(5)
      COMMON /BLK1/ NW,NZ,CV(700)
      WRITE (6,10) (ICH(I),I=1,10)
   10 FORMAT(1H1,10X,10A6)
```

```
      HHH = H(ILH)
      WRITE (6,20) H(ILH), XK, E, XMU, RZERO, DELTA(ILD)
   20 FORMAT (1H0,10X,22HRUNWAY CHARACTERISTICS / 11X,3HH =,F8.3,
     1 4H(IN),5X,3HK =,F8.1,11H(LBF/IN**3),5X,3HE =,F10.1 /
     2 11X,4HMU =,F6.3,5X,4HRO =,F8.2,4H(IN),5X,7HDELTA =,F7.3 )
      WRITE (6,30) XLA,XLB,XLC,XLD
   30 FORMAT(1H0,10X, 4HGEAR /
     1          1H ,10X,3HA =,F7.2,8H(IN) B =,F7.2,8H(IN) C =,F7.2,
     2          8H(IN) D =,F7.2,4H(IN) )
      WRITE (6,40) XNA,XNB,XNC,XND
   40 FORMAT(1H ,10X,4HNA =,F5.1,6X,4HNB =,F5.1,6X,4HNC =,F5.1,6X,4HND =
     1 , F5.1 )
      WRITE (6,50) A,PHIE
   50 FORMAT(1H0,10X,26HCONTACT AREA OF ONE TIRE =,F9.2,8H(SQ.IN.),
     1 10X, 6HPHIE =,F5.3)
      WRITE (6,60) P
   60 FORMAT(1H0, 10X,20HINFLATION PRESSURE =,F7.1,5H(PSI) )
      WRITE (6,70) G,B
   70 FORMAT(1H0, 10X,11HGEAR LOAD =,F10.1,5H(LBF),10H       B =,F8.1//)
      DO 500 INOG = 1,NOG
      GAMMA(INOG) = GAMMA(INOG)*DPR
      WRITE (6,90) INOG,GAMMA(INOG),PVMTST(INOG),YMN(INOG),SIGMA(INOG)
   90 FORMAT(1H ,10X,6HGAMMA(,I2,3H) =,F8.3,9H(DEG) N =,F7.2,5H MN =,
     1 F9.2,8H(IN.LB.) /
     2 1H ,10X,17HPAVEMENT STRESS =,F11.3,5H(PSI)//)
  500 CONTINUE
  310 IF (XOP3 - 0.5) 160, 160, 110
  110 WRITE (6,120)
  120 FORMAT (1H0,14X,1HZ,14X,1HW,10X,8HSUBTOTAL,10X,5HGAMMA)
      DO 150 J=1,NOG
      DO 150 I=1,NPT
      DISTY=RZERO * ZP(I,J)
      DISTX=RZERO * WP(I,J)
      STRESS = (RZERO*RZERO/10000.)*P*GPRINT(I,J)
      STRESS = 6.*STRESS/(HHH*HHH)
      WRITE(6,130) DISTY,DISTX,STRESS ,GAMMA(J)
  130 FORMAT ((10H          ,2(F10.4,5X),2(F10.3,5X)))
  150 CONTINUE
  160 CONTINUE
      DO 600 INOG = 1, NOG
  600 GAMMA(INOG) = GAMMA(INOG)*RPD
      RETURN
      END
C
@FOR,IS BLOCK
                    BLOCK DATA
C
C
      COMMON /BLK1/ NW,NZ,CV(700)
      DATA NW, NZ / 25, 25 /
      DATA (CV(J),J=1,175) /0.,.1,.2,.3,.4,.5,.6,.8,1.0,1.2,1.4,1.6,1.8,
     1  2.0,2.2,2.4,2.6,2.8,3.0,3.2,3.4,3.6,3.8,4.0,8.0,
     2  0.,.1,.2,.3,.4,.5,.6,.8,1.0,1.2,1.4,1.6,1.8,2.0,2.2,2.4,2.6,2.8,
     3  3.0,3.2,3.4,3.6,3.8,4.0,8.0,
     4 0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,
     5  0.,0.,0.,0.,
     6 0.,59.61,96.44,121.67,139.43,151.81,160.10,167.67,166.85,160.54,
     7  150.71,138.77,125.74,112.35,99.14,86.47,74.61,63.70,53.82,45.03,
     8  37.23,30.47,24.65,19.70,19.70,
     9 0.,104.57,175.97,226.34,262.29,287.66,304.92,321.48,321.23,
     A  309.93,291.52,268.79,243.78,217.95,192.40,167.84,144.80,123.58,
     B  104.35,87.18,72.35,58.88,47.55,37.91,37.91,
     C 0.,140.04,241.75,315.90,369.83,408.44,435.13,461.85,463.43,
     D  448.39,422.58,390.18,354.22,316.90,279.84,244.15,210.61,179.69,
     E  151.65,126.60,104.52,85.29,68.75,54.68,54.69,
     F 0.,169.01,296.76,392.57,463.64,515.33,551.63,589.36,593.86,
     G  576.22,544.14,503.13,457.21,409.31,361.58,315.50,272.13,232.10,
     H  195.77,163.30,134.67,109.73,88.28,70.05,70.05/
      DATA (CV(J),J=176,325) /0.,195.18,363.27,455.41,545.37,609.40,
     J  655.41,704.64,712.96,693.75,656.43,607.82,552.89,495.29,437.69,
     K  381.96,329.41,280.86,236.75,197.31,162.52,132.22,106.17,84.03,
     K  84.03,
     L 0.,213.63,382.94,515.16,616.59,692.58,747.58,808.45,821.26,
     M  801.36,759.75,704.47,641.42,574.95,508.25,443.57,382.47,325.96,
     N  274.58,228.61,188.05,152.73,122.37,96.58,96.58,
     O 0.,246.13,445.42,606.68,732.99,829.74,901.53,984.96,1007.91,
```

```
     P 988.70,960.95,874.88,798.08,716.24,633.53,552.95,476.58,405.78,
     Q 341.31,253.57,232.61,188.24,150.11,117.75,117.75,
     R 0.,270.36,496.03,676.25,821.93,935.68,1021.74,1125.54,1159.02,
     S 1142.32,1090.98,1016.97,929.35,834.99,738.97,644.67,555.59,
     T 472.54,396.80,328.91,268.96,216.78,171.96,133.97,133.97,
     U 0.,298.65,530.07,729.03,893.03,1017.39,1115.16,1236.65,1279.87,
     V 1265.56,1213.40,1135.69,1037.67,933.23,828.27,721.09,620.76,
     W 527.33,442.00,365.44,297.82,238.97,188.45,145.68,145.68,
     X 0.,302.51,557.45,769.25,942.15,1080.24,1187.41,1323.53,1375.63,
     Y 1365.74,1311.89,1228.14,1125.70,1013.25,897.30,782.99,673.55,
     Z 571.39,477.98,394.10,319.99,255.52,200.22,153.45,153.45/
      DATA (CV(J),J=326,475) /0.,313.02,578.23,799.85,981.92,1128.36,
     1 1242.95,1390.47,1450.14,1443.98,1389.98,1303.40,1196.05,1077.26,
     2 954.22,832.28,715.29,605.92,505.79,415.79,336.27,267.12,207.86,
     3 157.78,157.78,
     4 0.,320.96,593.65,823.03,1012.11,1166.98,1285.36,1442.14,1507.91,
     5 1504.67,1451.04,1362.45,1251.33,1127.56,998.78,870.73,747.60,
     6 632.29,526.59,431.55,347.56,274.58,212.01,159.27,159.27,
     7 0.,326.97,605.76,840.47,1034.86,1192.65,1317.41,1481.40,1552.01,
     8 1551.27,1498.00,1407.95,1293.94,1166.28,1032.90,899.94,771.82,
     9 651.67,541.43,442.28,354.85,278.52,213.34,158.47,158.47,
     A 0.,331.34,616.56,853.47,1051.83,1213.27,1341.39,1510.85,1585.19,
     B 1586.62,1533.67,1442.32,1326.06,1195.20,1058.29,921.39,789.25,
     C 665.20,551.29,448.81,358.24,279.59,212.31,155.74,155.74,
     D 0.,334.62,621.03,863.04,1064.33,1228.49,1359.08,1532.61,1609.74,
     E 1612.63,1559.70,1467.67,1347.63,1215.42,1075.53,936.69,801.13,
     F 673.94,557.29,451.95,359.04,278.43,209.49,151.65,151.65,
     G 0.,336.98,625.71,869.96,1073.37,1239.50,1371.88,1548.36,1627.49,
     H 1631.22,1578.58,1485.82,1366.34,1231.16,1088.96,946.42,808.51,
     I 678.83,559.65,452.41,357.66,275.49,205.33,146.53,146.53/
      DATA (CV(J),J=476,575) /0.,338.55,629.02,874.85,1079.76,1247.28,
     J 1380.92,1559.47,1639.98,1644.37,1591.60,1498.27,1377.61,1240.86,
     K 1096.82,952.30,812.36,680.71,559.69,450.81,354.64,271.28,
     L 200.23,140.72,140.72,
     M 0.,339.80,631.30,878.22,1084.15,1252.62,1387.11,1567.04,1648.43,
     N 1653.19,1600.37,1506.36,1384.71,1246.68,1101.16,955.06,815.53,
     O 680.34,557.59,447.43,350.48,266.24,194.48,134.53,134.53,
     P 0.,360.56,632.80,880.44,1087.04,1256.12,1391.16,1571.96,1653.86,
     Q 1658.75,1605.70,1511.13,1388.64,1249.56,1102.84,955.49,812.71,
     R 678.32,554.77,443.64,345.57,260.69,188.46,128.21,128.21,
     S 0.,341.02,633.72,881.80,1058.81,1258.26,1393.62,1574.90,1657.03,
     T 1661.89,1608.55,1513.45,1390.24,1250.29,1102.61,954.27,810.51,
     U 675.20,550.81,438.95,340.29,254.97,182.43,122.01,122.01/
      DATA (CV(J),J=576,675) /0.,341.28,634.23,882.55,1089.78,1259.41,
     V 1394.93,1576.40,1658.55,1663.25,1509.57,1515.97,1390.13,1249.46,
     W 1101.30,951.86,807.33,671.50,546.28,433.88,334.80,249.19,176.46,
     X 115.98,115.98,
     Y 0.,341.39,634.65,882.86,1090.16,1259.85,1395.41,1576.88,1658.93,
     Z 1663.35,1609.36,1515.27,1385.87,1247.59,1095.40,948.71,803.58,
     1 667.01,541.53,428.75,329.38,243.57,170.77,110.28,110.28,
     2 0.,341.39,634.45,882.86,1090.14,1259.81,1395.33,1576.67,1658.52,
     3 1662.68,1608.26,1511.76,1386.88,1245.10,1095.48,945.20,799.60,
     4 662.62,536.79,423.73,324.16,238.24,165.42,104.93,104.93,
     2 0.,341.39,634.45,882.86,1090.14,1259.81,1395.33,1576.67,1658.52,
     3 1662.68,1608.26,1511.76,1386.88,1245.10,1095.48,945.20,799.60,
     6 662.62,536.79,423.73,324.16,238.24,165.42,104.93,104.93/
      END
@FOR,IS ERN
C
C        C.C.ELLIS
C     **PROGRAM TO PRINT ERROR NO., JOB NO., ETC. OFF AND/OR ON-LINE.
C     **ARGUMENT DEFINITION
C       N1--ERROR NUMBER.
C           FIXED POINT INTEGER OF NOT MORE THAN 3 DIGITS.
C       N2--1 DIMENSIONAL ARRAY CONTAINING THE FOLLOWING.
C             N2(1)--PROCEDURE CODE.
C                    3 HOLLERITH CHARACTERS, LEFT ADJUSTED.
C             N2(2)--NAME OF SUBROUTINE
C                    6 HOLLERITH CHARACTERS OR LESS
C             N2(3)--OFF-LINE, ON-LINE PRINT OPTION
C                    N2(3)=1 PRINT OFF-LINE ONLY
C                    N2(3)=2 PRINT ON-LINE ONLY
C                    N2(3)=3 PRINT OFF-LINE AND ON-LINE
C             N2(4)--CARD NO. OPTION
C                    N2(4)=0 USE CARD NO. IN IS(4)
C                    N2(4) NOT =0 SET IS(4)=N2(4) AFTER CALL STATUS
```

```
C
C              N2(5)--ERN ENTRY COUNTER.
C                     N2(5) IS INCREMENTED BY 1 EACH TIME ERN IS ENTERED.
C                     N2(5) MUST BE INITIALIZED BY CALLING PROGRAM.
C          N3--STATEMENT NO. OR SOME OTHER NUMERIC TO INDICATE STATEMENT
C              FROM WHICH ERN WAS CALLED IN CALLING PROGRAM. N3 MUST BE A
C              FIXED POINT INTEGER OF NOT MORE THAN 5 DIGITS.
C
      SUBROUTINE ERN (N1, N2, N3)
C
      DIMENSION  IS(7), N2(5)
C
      DATA K2, K3 /6HL     , 6HP
C
      N2(5) = N2(5) + 1
      IF (N2(4)) 90,95,90
   90 IS(4) = N2(4)
   95 IF (IS(2) - 2) 100,150,150
  100 K1 = K2
      GO TO 190
  150 K1 = K3
  190 IF (N2(3) - 2) 200,300,200
  200 WRITE (6,900) N1, IS(1), K1, IS(3), IS(4), IS(5), IS(6), IS(7),
     1              N2(1), N2(2), N3
      IF (N2(3) - 1) 400,400,300
  300 PRINT 910, IS(1), K1, IS(3), N2(1)
  400 RETURN
  900 FORMAT (11H0ERROR NO. ,I3,12H.   JOB NO. ,I6,14H.   DECK TYPE ,A1,
     113H.   DECK NO. ,I3/19H CARD SEQUENCE NO. ,I5,14H.   LINE CTR. ,I3
     2,13H.   PAGE NO. ,I3/6H DATE ,A6,15H.   PROCEDURE ,A3,15H.   SUBR
     3OUTINE ,A6,15H STATEMENT NO. ,I5)
  910 FORMAT (18H ERROR IN JOB NO. ,I6,13H.  DECK TYPE ,A1,12H.  DECK NO
     1. ,I3,13H.  PROCEDURE ,A3)
      END
C
@FOR,IS GEOM
                           SUBROUTINE GEOM
C
C
      COMMON /ABLK/ A,AP(10),AM(10),AK(10),ASIG(10),AG(10)
      COMMON /BBLK/ BIGX,BIGY,B
      COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)
      COMMON /EBLK/ E,ER(100)
      COMMON /FBLK/ FG(10)
      COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)
      COMMON /HBLK/ H(10)
      COMMON /IBLK/ ICM(10),ILH,IPT,IT,ILD,ISG,INOG    ,KBP
      COMMON /NBLK/ NOG,NOM,NPT,NX,NY,NOD,NOSG
      COMMON /PBLK/ P,PI,PVMTST(10),PHIE
      COMMON /RBLK/ RPD,RZERO
      COMMON /SBLK/ SIGMA(10),S,SR
      COMMON /TBLK/ T,TR
      COMMON /WBLK/ WR,WR2
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOH,XNOD,XNOSG,XNT,XOP1,
     2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),
     3              XZERO,XZEROP
      COMMON /YBLK/ YMN(10),WP(100,10) ,YPZ(4097),YZERO,YZEROP
      COMMON /NSTA1/ KERN(5)
      COMMON /BLK1/ NW,NZ,CV(700)
      RZERO =SQRT( SQRT( (E*H(ILH)**3) / ( 12.*(1.-XMU**2)*XK ) ) )
      XLAR = XLA / RZERO
      XLBR = XLB / RZERO
      XLCR = XLC / RZERO
      XLDR = XLD / RZERO
      WR = (XNA-1.) * XLAR + XNA*(XNB-1.)*XLBR
      XLR= (XNC-1.) * XLCR + XNC*(XND-1.)*XLDR
      NX =INT((XNA*XNB+1.) / 2.0 + 0.0001)
      NY =INT((XNC*XND+1.) / 2.0 + 0.0001)
      WR2= WR/2.
      ER(2) = XLAR-XLBR
      XLR2  = XLR/2.
      ER(5) = XLCR-XLDR
      N1     =INT(XNB + 0.0001)
      DO 10 I = 1,NX
      ER(3) = I - 1
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      ER(I+50) = WR2 - ER(3)*XLBR - ER(2) * FLOAT ((I-1)/N1)
   10 CONTINUE
      N2 =INT(XND + 0.0001)
      DO 20 J = 1,NY
      ER(6) = J - 1
      ER(J+7) = XLR2 - ER(6)*XLDR - ER(5) * FLOAT ((J-1)/N2)
   20 CONTINUE
      NX1 = NX
      NY1 = NY
      IF ( ABS(ER(NX+50)) - 1.E-07 ) 30,30,40
   30 ER(NX+50) = 0.
      NX1 = NX - 1
   40 NPT = 0
      DO 60 J = 1,NY
      DO 50 I = 1,NX
      NPT = NPT + 1
      XPZ(NPT) = ER(I+50)
      YPZ(NPT) = ER(J+7 )
   50 CONTINUE
   60 CONTINUE
      IF ( ABS(ER(NY+7)) - 1.E-07 ) 70,70,75
   70 ER(NY+7) = 0.
      NY1 =NY-1
   75 IF ( NY1 ) 105,105,80
   80 DO 100 J = 1,NY1
      DO 90  I = 1,NX
      NPT = NPT + 1
      XPZ(NPT) = ER(I+50)
      YPZ(NPT) =-ER(J+7)
   90 CONTINUE
  100 CONTINUE
  105 IF (NX1) 123,123,107
  107 DO 120 J = 1,NY
      DO 110 I = 1,NX1
      NPT = NPT + 1
      XPZ(NPT) = -ER(I+50)
      YPZ(NPT) =   ER(J+7)
  110 CONTINUE
  120 CONTINUE
  123 IF (NX1) 145,145,125
  125 IF (NY1) 145,145,128
  128 DO 140 J = 1,NY1
      DO 130 I = 1,NX1
      NPT = NPT + 1
      XPZ(NPT) = -ER(I+50)
      YPZ(NPT) = -ER(J+7)
  130 CONTINUE
  140 CONTINUE
  145 IF (DP(1)-0.5) 170,170,150
  150 WRITE (6,160) RZERO,XLAR,XLBR,XLCR,XLDR,WR,XLR
  160 FORMAT (1H0,2X,32HRZERO,XLAR,XLBR,XLCR,XLDR,WR,XLR /
     1 1H ,10E12.4)
  170 IF (DP(2)-0.5) 200,200,180
  180 WRITE (6,185) NX,NY,NPT,NX1,NY1,ILH
  185 FORMAT (1H0,35H   NX    NY   NPT   NX1   NY1   ILH       / 1H ,10I5 )
      WRITE (6,190) ER , (XPZ(I),YPZ(I),I=1,NPT)
  190 FORMAT(1H0,2X,37HER(1)-ER(100),(XPZ(I),YPZ(I),I=1,NPT) /
     1   (1H , 10E12.4 ) )
  200 CONTINUE
      RETURN
      END
C
C
C
@FOR,IS OUTLNE
                    SUBROUTINE OUTLNE
C
C
      COMMON /ABLK/ A,AP(10),AM(10),AK(10),ASIG(10),AG(10)
      COMMON /BBLK/ BIGX,BIGY,B
      COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)
      COMMON /EBLK/ E,ER(100)
      COMMON /FBLK/ FG(10)
      COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)
      COMMON /HBLK/ H(10)
      COMMON /IBLK/ ICM(10),ILH,IPT,IT,ILD,ISG,INOG    ,KBP
```

```
      COMMON /NBLK/ NOG,NOM,NPT,NX,NY,NOD,NOSG
      COMMON /PBLK/ P,PI,PVMTST(10),PHIE
      COMMON /RBLK/ RPD,RZERO
      COMMON /SBLK/ SIGMA(10),S,SR
      COMMON /TBLK/ T,TR
      COMMON /WBLK/ WR,WR2
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOM,XNOD,XNOSG,XNT,XOP1,
     2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),
     3              XZERO,XZEROP
      COMMON /YBLK/ YMN(10),WP(100,10) ,YPZ(4097),YZERO,YZEROP
      COMMON /HSTAT/ KERN(5)
      COMMON /BLK1/ NW,NZ,CV(700)
      COMMON /UBLK/ UNUM(10,10),UMAX(10)
      SAM(PX)=(1.+(PHIE-1.)*PX**3)*(SIN(PI*PX/2.))/PHIE
      S = SQRT(A/(PI*PHIE))
      T = SQRT(A*PHIE/PI)
      SR= S/RZERO
      TR= T/RZERO
      ILD = 1
      IF (NOD .EQ. 0) GO TO 8
    7 DELTAR(ILD) = DELTA(ILD)/RZERO
    8 DO 50 K = 1,10
      PVMTST(K) = 0.0
      DO 51 J = 1,100
      GPRINT(J,K) = 0.0
   51 CONTINUE
   50 CONTINUE
      DO 305 INOG=1,NOG
      IF (XOP6 .GT.0.5) GO TO 10
      IF(ABS(GAMMA(INOG)).LT.0.0001) GO TO 20
      IF ( ABS(GAMMA(INOG) - 90.*RPD) .LT. 1.0E-7 ) GO TO 30
      GETAN = SIN(GAMMA(INOG))/COS(GAMMA(INOG))
      XZEROP = -(XPZ(1)+SR*GETAN/SQRT((GETAN)**2
     1 + PHIE**2)  + DELTAR(ILD)*COS(GAMMA(INOG)))
      YZEROP = -(YPZ(1)+TR*PHIE/SQRT((GETAN)**2 +
     1 PHIE**2) - DELTAR(ILD) * SIN(GAMMA(INOG)))
      GO TO 40
   10 XZEROP = BIGX/RZERO
      YZEROP = BIGY/RZERO
      GO TO 40
   20 XZEROP = -(DELTAR(ILD) + XPZ(NX))
      YZEROP = -(YPZ(1) + TR)
      GO TO 40
   30 XZEROP = -(XPZ(1) + SR)
      YZEROP = (DELTAR(ILD) + YPZ(NY))
   40 M =INT(B/4.0 + 0.99999999)
      KBP= 4*M
C                   START LOOP ON TIRE POINTS
      IE1=0
      IE2=0
      IE3=0
      IE4=0
      DO 250 IPT = 1,NPT
      XZERO=XPZ(IPT) + SR - XZEROP
      YZERO=YPZ(IPT) - YZEROP
      ZZERO=XZERO*COS(GAMMA(INOG))-YZERO*SIN(GAMMA(INOG))
      WZERO=XZERO*SIN(GAMMA(INOG))+YZERO*COS(GAMMA(INOG))
C                    START LOOP ON TIRE PERIMETER
      DO 200 J=1,KBP
      DM =FLOAT(M)
      DJ =FLOAT(J)
      IF(1.LE.J .AND. J.LE.M) GO TO 60
      IF(M+1.LE.J .AND. J.LE.2*M) GO TO 70
      IF(2*M+1.LE.J .AND. J.LE.3*M) GO TO 80
      PX = (4.*DM-DJ)/DM
      XI=XPZ(IPT)-XZEROP+SR*SQRT(1.-SAM(PX)**2)
      YI=YPZ(IPT)-YZEROP-TR*SAM(PX)
      GO TO 90
   60 PX = (DJ/DM)
      XI=XPZ(IPT)-XZEROP+SR*SQRT(1.-SAM(PX)**2)
      YI=YPZ(IPT)-YZEROP+TR*SAM(PX)
      GO TO 90
   70 PX = (2.*DM-DJ)/DM
      XI=XPZ(IPT)-XZEROP-SR*SQRT(1.-SAM(PX)**2)
      YI=YPZ(IPT)-YZEROP+TR*SAM(PX)
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```
      GO TO 90
   80 PX = (DJ-2.*DR)/DR
      XI=XPZ(IPT)-XZEROP-SR*SQRT(1.-SAM(PX)**2)
      YI=YPZ(IPT)-YZEROP-TR*SAM(PX)
   90 ZI=XI*COS(GAMMA(INOG))-YI*SIN(GAMMA(INOG))
      WI=XI*SIN(GAMMA(INOG))+YI*COS(GAMMA(INOG))
      ZINT = (ZI + ZZERO) / 2.0
      IF (ZINT .LT. 0.0) GO TO 110
      CALL TABINT (ZINT, TNA, WI, NZ, NW, CV, IND)
      IF (IND.LT.2) GO TO 100
      IF (ABS(WI) .LT. 1.0E-05) GO TO 100
      IE1=IE1+1
      IF(IE1.GT.2) GO TO 400
      WRITE (6,198) IND, ZINT, WI
  198 FORMAT (1H0,32HDATA IS OUTSIDE LIMITS OF TABLE. / 10H     IND =,I2
     1 ,5X,6HZINT =,E15.7,5X,4HWI =,E15.7)
  400 CONTINUE
      TNA = 0.0
  100 CALL TABINT (ZINT, TNB, WZERO, NZ, NW, CV, IND)
      IF (IND .LT. 2) GO TO 130
      IF (ABS(WZERO) .LT. 1.0E-05) GO TO 130
      IE2=IE2+1
      IF(IE2.GT.2) GO TO 500
      WRITE (6,199) IND, ZINT, WZERO
  199 FORMAT (1H0,32HDATA IS OUTSIDE LIMITS OF TABLE. / 10H     IND =,I2
     1 ,5X,6HZINT =,E15.7,5X,7HWZERO =,E15.7)
  500 CONTINUE
      TNB=0.0
      GO TO 130
  110 AZINT = -ZINT
      CALL TABINT (AZINT, TSA, WI, NZ, NW, CV, IND)
      TNA = -TSA
      IF (IND.LT.2) GO TO 120
      IF (ABS(WI) .LT. 1.0E-05) GO TO 120
      IE3=IE3+1
      IF(IE3.GT.2) GO TO 600
      WRITE (6,191) IND, AZINT, WI
  191 FORMAT (1H0,32HDATA IS OUTSIDE LIMITS OF TABLE. / 10H     IND =,I2
     1 ,5X,7HAZINT =,E15.7,5X,4HWI =,E15.7)
  600 CONTINUE
      TNA = 0.0
  120 CALL TABINT (AZINT, TSB, WZERO, NZ, NW, CV, IND)
      TNB = -TSB
      IF (IND .LT. 2) GO TO 130
      IF (ABS(WZERO) .LT. 1.0E-05) GO TO 130
      IE4=IE4+1
      IF(IE4.GT.2) GO TO 700
      WRITE (6,192) IND, AZINT, WZERO
  192 FORMAT (1H0,32HDATA IS OUTSIDE LIMITS OF TABLE. /
     1 4X,5HIND =,I2,5X,7HAZINT =,E15.7,5X,7HWZERO =,E15.7)
  700 CONTINUE
      TNB = 0.0
  130 TANS=TNA-TNB
      IF (DP(3) - 0.5) 150, 150, 140
  140 WRITE (6,95) XI, YI, ZI, WI, XZERO, YZERO, ZZERO, WZERO,
     1 ZINT, GAMMA(INOG), TNA, TNB
   95 FORMAT (1H0,6X,2HXI,13X,2HYI,13X,2HZI,13X,2HWI/     4(E15.7)//
     1   5X,5HXZERO,10X,5HYZERO,10X,5HZZERO,10X,5HWZERO/     4(E15.7)//
     2 6X,4HZINT,6X,11HGAMMA(INOG),9X,3HTNA,12X,3HTNB/ 4(E15.7))
  150 GPRINT(IPT,INOG) = GPRINT(IPT,INOG) + TANS
  197 XZERO=XI
      YZERO=YI
      ZZERO=ZI
      WZERO=WI
  200 CONTINUE
      PVMTST(INOG) = PVMTST(INOG) + GPRINT(IPT,INOG)
      IF (XOPS - 0.5) 250,250,230
  230 ZP(IPT,INOG)= COS(GAMMA(INOG))*(XPZ(IPT)-XZEROP) - SIN(GAMMA(INOG)
     1 )*(YPZ(IPT)-YZEROP)
      WP(IPT,INOG)= SIN(GAMMA(INOG))*(XPZ(IPT)-XZEROP) + COS(GAMMA(INOG)
     1 )*(YPZ(IPT)-YZEROP)
  250 CONTINUE
      YMN(INOG)=RZERO*RZERO/10000.0*P*PVMTST(INOG)
      SIGMA(INOG)=6.0*YMN(INOG)/H(ILH)**2
      IF (NOSG .EQ. 0) GO TO 335
      DO 280 ISG=1,NOSG
```

```
      AP(ISG)= AG(ISG)/(XNT*A)
      AH(ISG)=RZERO*SQRT(ABS((6.0*AP(ISG)*PVMTST(INOG))/(10000.*ASIG
     1 (ISG))))
      AK(ISG) = (34.1/AH(ISG))*(10.0*AH(ISG)/RZERO)**6
  280 CONTINUE
  290 CALL CURVE
  305 CONTINUE
      CALL FINISH
      DO  330  INOG=1,NOG
  330 UNUM(ILD,INOG)=PVMTST(INOG)
      IF (NOD .EQ. 0) GO TO 320
      ILD = ILD + 1
      IF (ILD .LE. NOD) GO TO 7
  320 RETURN
      END
C
@FOR,IS CURVE
                     SUBROUTINE CURVE
C
C
      COMMON /ABLK/ A,AP(10),AH(10),AK(10),ASIG(10),AG(10)
      COMMON /BBLK/ BIGX,BIGY,B
      COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)
      COMMON /EBLK/ E,ER(100)
      COMMON /FBLK/ FG(10)
      COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)
      COMMON /HBLK/ H(10)
      COMMON /IBLK/ ICR(10),ILH,IPT,IT,ILD,ISG,INOG,  KBP
      COMMON /NBLK/ NOG,NOH,NPT,NX,NY,NOD,NOSG
      COMMON /PBLK/ P,PI,PVMTST(10),PHIE
      COMMON /RBLK/ RPD,RZERO
      COMMON /SBLK/ SIGMA(10),S,SR
      COMMON /TBLK/ T,TR
      COMMON /WBLK/ WR,WR2
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOH,XNOD,XNOSG,XNT,XOP1,
     2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),
     3              XZERO,XZEROP
      COMMON /YBLK/ YMN(10),WP(100,10) ,YPZ(4097),YZERO,YZEROP
      COMMON /H51A1/ KERN(5)
      COMMON /BLK1/ NW,NZ,CV(700)
      GAMMA(INOG) = GAMMA(INOG) * DPR
      WRITE (6,10) E, XMU
   10 FORMAT (1H1,4X, 3HE =,F10.1,5X, 5HXMU =,F6.3)
      WRITE (6,30) XLA,XLB,XLC,XLD
   30 FORMAT(1H0,10X, 4HGEAP /
     1         1H ,10X,3HA =,F7.2,8H(IN) B =,F7.2,8H(IN) C =,F7.2,
     2         8H(IN) D =,F7.2 )
      WRITE (6,40) XNA,XNB,XNC,XND
   40 FORMAT(1H ,10X,4HNA =,F5.1,6X,4HNB =,F5.1,6X,4HNC =,F5.1,6X,4HND =
     1 , F5.1 )
      WRITE (6,50) A,PHIE
   50 FORMAT(1H0,10X,26HCONTACT AREA OF ONE TIRE =,F9.2,8H(SQ.IN.),
     1 10X, 6HPHIE =,F5.3)
      WRITE (6,60) H(ILH), GAMMA(INOG), DELTA(ILD), PVMTST(INOG)
   60 FORMAT (1H0,4X, 3HH =,F8.3,5X, 7HGAMMA =,F8.3,5X, 7HDELTA =,F8.3,
     1  5X, 3HN= ,F11.3)
      DO 100 ISG=1,NOSG
      WRITE (6,70) ASIG(ISG), AG(ISG), AP(ISG), AH(ISG), AK(ISG)
   70 FORMAT (1H0,2X, 7HSIGMA =,F 8.3,2X, 3HG =,F 8.1,2X, 3HP =F7.1,2X,
     1   3HH =F8.3,2X,3HK =,F8.1)
  100 CONTINUE
      GAMMA(INOG) = GAMMA(INOG) * RPD
      RETURN
      END
C
@FOR,IS TABINT
      SUBROUTINE TABINT (X,Y,Z,NX,NZ,CV,IND)
      DIMENSION CV(700)
      IND = 1
      DO 100 I=26,50
      IF(Z.LT.CV(26))GO TO 200
      IF(Z.LT.CV(I)) GO TO 300
  100 CONTINUE
      IND = 5
      IS = 49
```

```
      IL = 50
      GO TO 250
  200 IND = 4
      IS = 26
      IL = 27
      GO TO 250
  300 IL = I
      IS = I-1
 250  DO 800 J = 1,25
      IF(X.LT.CV(1)) GO TO 900
      IF(X.LT.CV(J)) GO TO 1000
  800 CONTINUE
      IND = 3
      JS = 24
      JL = 25
      GO TO 950
  900 JS = 1
      JL = 2
      IND=2
      GO TO 950
 1000 JL = J
      JS = J-1
  950 CONTINUE
      IZ =IS-25
      IQ =IL-25
      K1=50+(IZ-1)*25+JS
      Y1S=CV(K1)
      K2=K1+1
      Y1L=CV(K2)
      K3=50+(IQ-1)*25+JS
      Y2S=CV(K3)
      K4=K3+1
      Y2L=CV(K4)
      IF(IND.GT.1) GO TO 1100
 1010 Y1=(((Y1L-Y1S)/(CV(JL)-CV(JS)))*(X-CV(JS)))+Y1S
      Y2=(((Y2L-Y2S)/(CV(JL)-CV(JS)))*(X-CV(JS)))+Y2S
      IF(IND.GT.1) GO TO 1050
 1020 Y=(((Y2-Y1)/(CV(IL)-CV(IS)))*(Z-CV(IS)))+Y1
      IF(IND.GT.1) GO TO 1070
      GO TO 2000
 1100 IF(X.LT.CV(1))GO TO 1010
      IF(X.GT.CV(25))GO TO 1040
      GO TO 1010
 1040 Y1= (((Y1L-Y1S)/(CV(JL)-CV(JS)))*(X-CV(JL)))+Y1L
      Y2= (((Y2L-Y2S)/(CV(JL)-CV(JS)))*(X-CV(JL)))+Y2L
 1050 IF(Z.LT.CV(26)) GO TO 1020
      IF(Z.GT.CV(50)) GO TO 1060
      GO TO 1020
 1060 Y = (((Y2-Y1)/(CV(IL)-CV(IS)))*(Z-CV(IL)))+Y2
 1070 CONTINUE
 2000 CONTINUE
      RETURN
      END
C
C
C
@FOR,IS PROBRD
                               SUBROUTINE PROBRD
C
C
      COMMON /ABLK/ A,AP(10),AM(10),AK(10),ASIG(10),AG(10)
      COMMON /BBLK/ BIGX,BIGY,B
      COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)
      COMMON /EBLK/ E,ER(100)
      COMMON /FBLK/ FG(10)
      COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)
      COMMON /HBLK/ H(10)
      COMMON /IBLK/ ICM(10),ILH,IPT,IT,ILD,ISG,INOG    ,KBP
      COMMON /NBLK/ NOG,NOH,NPT,NX,NY,NOD,NOSG
      COMMON /PBLK/ P,PI,PVATST(10),PHIE
      COMMON /RBLK/ RPD,RZERO
      COMMON /SBLK/ SIGMA(10),S,SR
      COMMON /TBLK/ T,TR
      COMMON /WBLK/ WR,WR2
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOH,XNOD,XNOSG,XNT,XOP1,
```

```
     2                  XOP2,XOP3,XOP4,XOP5,XOP5,ZP(100,10) ,XPZ(4097),
     3                  XZERO,XZEROP
      COMMON /YBLK/ YMN(10),WP(100,10) ,YPZ(4097),YZERO,YZEROP
      COMMON /HS1A1/ KERN(5)
      COMMON /BLK1/ NW,NZ,CV(700)
      COMMON /MPJ/  AO,A1,A2,ALPHA,AMR,BN(10),CVDDF(10),CVK,IG,NON,
     1              JOPPL1,JOPPL2
      DATA KNAME/6HPROBRD/
      KERN(2) = KNAME
      PI = 3.1415927
      DPR= 180.0/PI
      RPD= PI/180.0
      READ(5,10,END=999)(ICM(I),I=1,10)
10    FORMAT(10A6)
      DP(1)=0.
      WRITE (6,20)
   20 FORMAT(1H1 ,19X,35HP R O B L E M   I N P U T   D A T A    )
      WRITE (6,30) (ICM(I),I=1,10)
   30 FORMAT (1H0, 11X, 10A6 )
15    READ(5,40,END=999)G,P,A,XNOG
   17  CONTINUE
40    FORMAT(4F10.0)
  211 IF (XNOG) 220, 220, 225
  220 CALL ERN (03, KERN, 220)
      GO TO 8990
  225 IF (XNOG - 10.) 230, 230, 215
  215 CALL ERN (04, KERN, 215)
      GO TO 8990
  230 IF (P.LT.0.0001.AND.A.LT.0.0001) GO TO 240
      WRITE (6,45)
   45 FORMAT (1H0,3X,11HK(LB/CU IN),5X,5HG(LB),8X,6HP(PSI),6X,8HA(SQ IN)
     1,4X,12HNO. GAMMA(I) )
      WRITE (6,50) XK, G, P, A, XNOG
   50 FORMAT (1H ,5X,F7.1,5X,F10.1,5X,F7.1,5X,F9.1,5X,F4.1)
      GO TO 245
  240 CALL ERN(05, KERN, 240)
      GO TO 8990
   41 FORMAT(6F10.0)
  245 READ (5,41) XLA, XLB, XLC, XLD, XNOD, XNOSG
      IF (XNOD) 250, 255, 255
  250 CALL ERN (06, KERN, 250)
      GO TO 8990
  255 IF (XNOD - 10.) 265, 265, 260
  260 CALL ERN (07, KERN, 260)
      GO TO 8990
  265 IF (XNOSG) 270, 275, 275
  270 CALL ERN (08, KERN, 270)
      GO TO 8990
  275 IF (XNOSG - 10.) 285, 285, 280
  280 CALL ERN (09, KERN, 280)
      GO TO 8990
  285 WRITE (6,60)
   60 FORMAT (1H0,3X,50HSMALL A(IN)  SMALL B(IN)  SMALL C(IN)  SMALL D(I
     1N),22H NO. DELTA(I) NO.(S,G) )
      NOG =INT(XNOG + 0.0001)
      NOD=INT(XNOD+0.0001)
      NOSG=INT(XNOSG+0.0001)
      WRITE (6,65) XLA,XLB,XLC,XLD,XNOD,XNOSG
   65 FORMAT (1H ,5X,F7.2,3(6X,F7.2),7X,F4.1,5X,F4.1)
   43 FORMAT(5F10.0)
      READ(5,43)XNA,XNB,XNC,XND,PHIE
      XOP1=0.
      IF (PHIE.LT.0.0001) PHIE = 5./3.
      WRITE (6,70)           XNA,XNB,XNC,XND,PHIE
   70 FORMAT (1H0,5X,3HXNA,9X,3HXNB,9X,3HXNC,9X,3HXND,9X,6HPHI(E)/
     11H ,4X,F4.1,4(8X,F4.1))
      READ (5,43) B, XOP3, BIGX, BIGY, XOP6
      WRITE (6,75) B, XOP3, BIGX, BIGY, XOP6
   75 FORMAT (1H0,8X,1HB,6X,9HPRINT OPT,3X,5HBIG X,7X,5HBIG Y,6X,
     18HCOOR OPT /1H ,5X,F7.1,5X,F4.1,2(5X,F7.2),7X,F4.1 )
42     FORMAT(10F6.0)
      READ (5,42) (GAMMA(I), I=1,NOG)
      WRITE (6,85) (GAMMA(I), I=1,NOG)
   85 FORMAT (1H0,27X,14HGAMMA(I) (DEG)/
     11H ,2X,6(F7.2,4X) )
      IF (NOD.EQ.0) GO TO 97
```

```
      READ (5,42) (DELTA(I), I=1,NOD)
      WRITE (6,90) (DELTA(I), I=1,NOD)
   90 FORMAT (1H0,30X,8HDELTA(I) /
     11H ,2X,6(F7.2,4X) )
   97 IF (NOSG .EQ. 0) GO TO 99
      READ (5,92) (ASIG(I), AG(I), I=1,NOSG)
 92   FORMAT(8F10.0)
      WRITE (6,95) ( ASIG(I),AG(I), I=1,NOSG)
   95 FORMAT (1H0,6X,7HASIG(I),9X,5HAG(I) /    2(5X,F10.1)/)
   99 DO 300 I = 1,NOG
      GAMMA(I) = GAMMA(I)*RPD
  300 CONTINUE
      IF (DP(1)-0.5) 340, 340, 325
  325 READ (5,330) (DP(I), I=1,31)
  330 FORMAT (31F2.0)
      WRITE (6,335) (DP(I), I=1,31)
  335 FORMAT (1H0, 5HDUMPS, 31F4.1)
  340 XNT=XNA*XNB*XNC*XND
      IF (XNT - 4096.001) 410, 400, 400
  400 CALL ERN (10, KERN, 400)
      GO TO 8990
  410 IF (A) 490, 500, 490
  490 P = G/(A*XNT)
      GO TO 510
  500 A = G/(P*XNT)
  510 G = A*P*XNT
      GO TO 540
 8990 CONTINUE
      GO TO 15
 999  CALL EXIT
  540 RETURN
      END
C
C
@FOR,IS KCOMP
      SUBROUTINE KCOMP
C
C
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOM,XNOD,XNOSG,XNT,XOP1,
     2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),
     3              XZERO,XZEROP
      COMMON /MPJ/  A0,A1,A2,ALPHA,AMR,BN(10),CVDDF(10),CVK,IG,NON,
     1              JOPPL1,JOPPL2
 1004 FORMAT(1H1,16X,'MULTI-LAYERED SYSTEM ANALYSIS'//)
 1005 FORMAT(6X,'SUBGRADE:')
 1006 FORMAT(14X,'MODULUS OF SUBGRADE REACTION = ',F7.2,' PCI',5X,
     1'CV = 'F7.4/)
 1007 FORMAT(5X'BASE:')
 1008 FORMAT(14X,'TYPE: WELL-GRADED CRUSHED MATERIALS'/)
 1009 FORMAT(14X,'TYPE: NATURAL SAND AND GRAVEL (PI<8)'/)
 1011 FORMAT(14X,'TYPE: STABILIZED'/)
 1012 FORMAT(14X,'HEIGHT = ',F5.2,' INCHES',8X,'CV = 'F7.4/)
 1013 FORMAT(14X,'MODULUS = ',F10.2,' PSI',5X,'CV = ',F7.4//)
 1014 FORMAT(1H0,2X,'COMPOSITE:')
 1015 FORMAT(5X,'LAYER  :')
 1016 FORMAT(5X,'SUBBASE:')
  701 FORMAT(I2,4F10.4)
  401 FORMAT(1H0,20X,'A MAXIMUM DESIGN K OF 500 PCI IS USED')
      WRITE(6,1004)
      WRITE(6,1005)
      WRITE(6,1006)XK,CVK
      DO 700 I=1,NOLBP
      READ(5,701)MATOPT,HB,CVHB,EB,CVEB
      IF(MATOPT.EQ.1)GO TO 101
      IF(MATOPT.EQ.2)GO TO 102
      IF(MATOPT.EQ.3)GO TO 103
  101 Y=ALOG10(XK)
      Y2=(0.08901252+(2.*(-0.0004425196)*HB)+(-0.02901488*Y))
      Y3=(HB**2.)*(Y2**2.)*(5.3018981)*(CVHB**2.)
      Y4=(0.9640174)+(-0.2561922*Y)+(-0.012601*HB)
      Y5=(Y4**2.)*(5.3018981)*(CVK**2.)
      CVKC=Y3+Y5
      CVK=SQRT(CVKC)
      Y1=-1.251182+(2.219232*Y)+(-0.2949522*(Y**2.))
     1+(0.08901252*HB)+(-0.0004425194*(HB**2.))
```

```
     2*(-0.02901488*MB*Y)
      XK=10.**Y1
      GO TO 195
  102 Z=ALOG10(XK)
      Z2=(0.08554373+(2.*(-0.0002574619)*MB)+(-0.03050173*Z))
      Z3=(MB**2.)*(Z2**2.)*(5.3018981)*(CVMB**2.)
      Z4=0.9829852+(-0.2617702*Z)+(-0.013246*MB)
      Z5=(Z4**2.)*(5.3108981)*(CVK**2.)
      CVKC=Z3+Z5
      CVK=SQRT(CVKC)
      Z1=-1.296084+(2.263407*Z)+(-0.3013741*(Z**2.))
     1+(0.08554373*MB)+(-0.0002574619*(MB**2.))
     2+(-0.03050173*MB*Z)
      XK=10.**Z1
      GO TO 195
  103 W=ALOG10(XK)
      W2=0.0544761+(-0.0016948*MB)+(-0.4409096E-2*W)
     1+(-0.2465638E-8*EB)
      W3=(MB**2.)*(W2**2.)*(5.3018981)*(CVMB**2.)
      W4=(0.7254749E-6)+(-0.3874586E-12*EB)+(-0.4601653E-7*W)
     1+(-0.2465638E-8*MB)
      W5=(EB**2.)*(W4**2.)*(5.3018981)*(CVEB**2.)
      W6=0.4465112+(-0.1914846E-2*MB)+(-0.1998473E-7*EB)
      W7=(W6**2.)*(5.3018981)*(CVK**2.)
      CVKC=W3+W5+W7
      CVK=SQRT(CVKC)
      W1=-0.1578667+(1.02813*W)+(0.0544761*MB)+(-0.8473852E-3*(MB**2.))
     1+(0.7254749E-6*EB)+(-0.1937293E-12*(EB**2.))
     2+(-0.4409096E-2*MB*W)+(-0.4601653E-7*EB*W)
     3+(-0.2465638E-8*EB*MB)
      XK=10.**W1
      GO TO 195
  195 CONTINUE
      IF(NOLBP.EQ.1)GO TO 201
      IF(NOLBP.EQ.2)GO TO 202
  201 WRITE(6,1007)
      IF(MATOPT.EQ.1)GO TO 203
      IF(MATOPT.EQ.2)GO TO 204
      IF(MATOPT.EQ.3)GO TO 205
  203 WRITE(6,1008)
      GO TO 206
  204 WRITE(6,1009)
      GO TO 206
  205 WRITE(6,1011)
      GO TO 206
  206 CONTINUE
      WRITE(6,1012)MB,CVMB
      IF(MATOPT.LT.2.5)GO TO 207
      WRITE(6,1013)EB,CVEB
  207 CONTINUE
      WRITE(6,1014)
      WRITE(6,1015)
      WRITE(6,1006)XK,CVK
      IF(XK.LE.500.)GO TO 400
      XK=500.
      WRITE(6,401)
  400 CONTINUE
      GO TO 899
  202 IF(I.EQ.1)GO TO 300
      IF(I.EQ.2)GO TO 301
  300 WRITE(6,1016)
      IF(MATOPT.EQ.1)GO TO 302
      IF(MATOPT.EQ.2)GO TO 303
      IF(MATOPT.EQ.3)GO TO 304
  302 WRITE(6,1008)
      GO TO 305
  303 WRITE(6,1009)
      GO TO 305
  304 WRITE(6,1011)
      GO TO 305
  305 CONTINUE
      WRITE(6,1012)MB,CVMB
      IF(MATOPT.LT.2.5)GO TO 306
      WRITE(6,1013)EB,CVEB
  306 CONTINUE
      GO TO 700
```

```
  301 WRITE(6,1007)
      IF(MATOPT.EQ.1)GO TO 307
      IF(MATOPT.EQ.2)GO TO 308
      IF(MATOPT.EQ.3)GO TO 309
  307 WRITE(6,1008)
      GO TO 310
  308 WRITE(6,1009)
      GO TO 310
  309 WRITE(6,1011)
      GO TO 310
  310 CONTINUE
      WRITE(6,1012)HB,CVHB
      IF(MATOPT.LT.2.5)GO TO 311
      WRITE(6,1013)EB,CVEB
  311 CONTINUE
      WRITE(6,1014)
      WRITE(6,1015)
      WRITE(6,1006)XK,CVK
      IF(XK.LE.500)GO TO 403
      XK=500.
      WRITE(6,401)
  403 CONTINUE
  899 CONTINUE
  700 CONTINUE
      RETURN
      END
@FOR,IS STORPL
C
C
C
C
      SUBROUTINE STORPL
C
C
C
C
      COMMON /ABLK/ A,AP(10),AH(10),AK(10),ASIG(10),AG(10)
      COMMON /BBLK/ BIGX,BIGY,B
      COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)
      COMMON /EBLK/ E,ER(100)
      COMMON /FBLK/ FG(10)
      COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)
      COMMON /HBLK/ H(10)
      COMMON /IBLK/ ICR(10),ILH,IPT,IT,ILD,ISG,INOG    ,KBP
      COMMON /NBLK/ NOG,NOH,NPT,NX,NY,NOD,NOSG
      COMMON /PBLK/ P,PI,PVMTST(10),PHIE
      COMMON /RBLK/ RPD,RZERO
      COMMON /SRLK/ SIGMA(10),S,SR
      COMMON /TBLK/ T,TR
      COMMON /WBLK/ WR,WR2
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOH,XNOD,XNOSG,XNT,XOP1,
     2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),
     3              XZERO,XZEROP
      COMMON /YBLK/ YMN(10),WP(100,10) ,YPZ(4097),YZERO,YZEROP
      COMMON /HS1A1/ KERN(5)
      COMMON /BLK1/ NW,NZ,CV(700)
      COMMON /UBLK/ UNUM(10,10),UMAX(10)
      COMMON /MPJ/  A0,A1,A2,ALPHA,AMR,BN(10),CVDDF(10),CVK,IG,NON,
     1              JOPPL1,JOPPL2
      DIMENSION DDF(10),PR(10)
      READ(5,101)(BN(I),I=1,NON)
  101 FORMAT(10F8.0)
      WRITE(6,103)
  103 FORMAT(//,9X,'H',10X,'RELIA',7X,'DF',15X,'N',/)
      NOH2=NOH+2
      IF(JOPPL2.LT.1)GO TO 71
      CALL PLOTMJ(H,PR,-1,NOH2)
   71 CONTINUE
      DO 50 II=1,NON
      DO 40 JJ=1,NOH
      SIG=UMAX(JJ)/H(JJ)/H(JJ)
      DF=AMR/ALPHA/SIG
      IF(XK.LT.250.)GO TO 31
      IF(XK.LT.350.)GO TO 33
      IF(XK.LT.450.)GO TO 35
```

```
      IF(BN(II).GT.5000.)GO TO 37
      DDF(II)=0.1*ALOG10(BN(II))+0.5401
      GO TO 38
   37 DDF(II)=0.246*ALOG10(BN(II))
      GO TO 38
   35 IF(BN(II).GT.5000.)GO TO 36
      DDF(II)=0.1186*ALOG10(BN(II))+0.6613
      GO TO 38
   36 DDF(II)=0.2767*ALOG10(BN(II))+0.0565
      GO TO 38
   33 IF(BN(II).GT.5000.)GO TO 34
      DDF(II)=0.1371*ALOG10(BN(II))+0.7029
      GO TO 38
   34 DDF(II)=0.3151*ALOG10(BN(II))+0.0465
      GO TO 38
   31 IF(BN(II).GT.5000.)GO TO 32
      DDF(II)=0.1445*ALOG10(BN(II))+0.7655
      GO TO 38
   32 DDF(II)=0.3382*ALOG10(BN(II))+0.049
   38 CONTINUE
      XY = (DDF(II)-DF)/((CVDDF(JJ)*DF)
      XX=XY
      IF(XX.LT.0.)XX=-XX
      ZX=EXP(-XX*XX/2.)/2.5066283
      TX=1./(1.+0.2316419*XX)
      PX=1.-ZX*TX*(0.31938153+TX*(-0.356563782+
     &          TX*(1.781477937+TX*(-1.821255978+TX*1.330274429))))
      PR(JJ)=1.-PX
      IF(XY.LT.0.)PR(JJ)=PX
   40 CONTINUE
      WRITE(6,102)((H(JJ),PR(JJ),DDF(II),BN(II)),JJ=1,NOH)
  102 FORMAT(F12.2,2F12.4,F15.0)
      IF(JOPPL2.LT.1)GO TO 72
      CALL PLOTMJ(H,PR,1,NOH2)
   72 CONTINUE
   50 CONTINUE
      RETURN
      END
@FOR,IS PLOTUZ
      SUBROUTINE PLOTUZ(X,Y,NN)
      DIMENSION X(33),Y(33)
      IF(NN) 10,10,20
   10 DO 11 I=3,33
      X(I)=0.
   11 Y(I)=0.
      X(1)=0.001
      X(2)=6.999
      Y(1)=0.001
      Y(2)=0.9999
      CALL PLOTC(1.0,2.0,-3)
      CALL SCALE(X,7.0,31,1)
      CALL SCALE(Y,5.0,31,1)
      CALL AXIS(0.0,0.0,13HLOG COVERAGES,-13,7.0,0.0,X(32),X(33))
      CALL AXIS(0.0,0.0,11HRELIABILITY,11,5.0,90.0,Y(32),Y(33))
      RETURN
   20 CALL PLOTC(0.0,0.0,3)
      CALL LINE(X,Y,31,1,0,0)
      RETURN
      END
@FOR,IS PLOTMJ
C
C
C
C
      SUBROUTINE PLOTMJ(X,Y,NN,NOH2)
C
C
C
C
      DIMENSION X(NOH2),Y(NOH2)
      NOH=NOH2-2
      NOH1=NOH2-1
      IF(NN)10,10,20
   10 DO 11 I=1,NOH2
   11 Y(I)=0.
      Y(1)=0.001
```

```
      Y(2)=3.9999
      CALL PAGEUP
      CALL PLOTC(1.0,2.0,-3)
      CALL SCALE(X,7.0,NOM,1)
      CALL SCALE(Y,5.0,NOM,1)
      CALL AXIS(0.0,0.0,14HSLAB THICKNESS,-14,7.0,0.0,X(NOM1),X(NOM2))
      CALL AXIS(0.0,0.0,11HRELIABILITY,11,5.0,90.0,Y(NOM1),Y(NOM2))
      RETURN
   20 CALL PLOTC(0.0,0.0,3)
      CALL LINE(X,Y,NOM,1,0,0)
      RETURN
      END
@FOR,IS GJR
      SUBROUTINE GJR(A,NC,NR,N,MC,S,JC,V)
      DIMENSION A(NR,NC),JC(1),V(2)
C              ----------------------------------------------------------
C               JC IS THE PERMUTATION VECTOR
C              KD IS THE OPTION KEY FOR DETERMINANT EVALUATION
C              KI IS THE OPTION KEY FOR MATRIX INVERSION
C              L IS THE COLUMN CONTROL FOR AX=B
C              M IS THE COLUMN CONTOL FOR MATRIX INVERSION
C              ----------------------------------------------------------
C              INITIALIZATION
C              ----------------------------------------------------------
      IW=V(1)
      M=1
      S=1.
      L=N+(MC-N)*(IW/4)
      KD=2-MOD(IW/2,2)
      IF(KD.EQ.1) V(2)=0.
      KI=2-MOD(IW,2)
      GO TO (5,20),KI
C              ----------------------------------------------------------
C                  INITIALIZE JC FOR INVERSION
C              ----------------------------------------------------------
5     DO 10  I=1,N
10    JC(I)=I
C              ----------------------------------------------------------
C              SEARCH FOR PIVOT ROW
C              ----------------------------------------------------------
20    DO 91  I=1,N
      GO TO (22,21),KI
21    M=I
22    IF (I.:D.N)  GO TO 60
      X=-1.
      DO 30  J=I,N
      IF (X.GT.ABS(A(J,I)))  GO TO 30
      X=ABS(A(J,I))
      K=J
30    CONTINUE
      IF(K.EQ.I) GO TO 60
      S=-S
      V(1)=-V(1)
      GO TO (35,40),KI
 35   MU=JC(I)
      JC(I)=JC(K)
      JC(K)=MU
C              ----------------------------------------------------------
C              INTERCHANGE ROW I AND ROW K
C              ----------------------------------------------------------
40    DO 50  J=M,L
      X=A(I,J)
      A(I,J)=A(K,J)
50    A(K,J)=X
C              ----------------------------------------------------------
C              TEST FOR SINGULARITY
C              ----------------------------------------------------------
   60 IF (ABS(A(I,I)).GT.1.E-7) GO TO 70
C              ----------------------------------------------------------
C              MATRIX IS SINGULAR
C              ----------------------------------------------------------
      IF(KD.EQ.1) V(1)=0.
      JC(1)=I-1
      RETURN 6
70    GO TO (71,72),KD
C              ----------------------------------------------------------
```

```
C                  COMPUTE THE DETERMINANT
C                  ---------------------------------------------------------
71      IF(A(I,I).LT.0.) S=-S
        V(2)=V(2)+ALOG(ABS(A(I,I)))
72      X=A(I,I)
        A(I,I)=1.
C                  ---------------------------------------------------------
C                  REDUCTION OF THE I-TH ROW
C                  ---------------------------------------------------------
         DO  80 J=1,L
        A(I,J)=A(I,J)/X
C                  ---------------------------------------------------------
C                  TEST OVERFLOW SWITCH. IF ON
C                  RETURN NEGATIVE VALUE OF I IN JC(1)
C                  ---------------------------------------------------------
        IF(ABS(A(I,J)).GT.10**75) IFL=1
        IF(IFL.EQ.1) GO TO 150
80      CONTINUE
C                  ---------------------------------------------------------
C                  REDUCTION OF ALL REMAINING ROWS
C                  ---------------------------------------------------------
        DO 91  K=1,N
        IF (K.EQ.I)  GO TO 91
        X=A(K,I)
        A(K,I)=0.
        DO 90  J =M,L
        A(K,J)=A(K,J)-X*A(I,J)
C                  ---------------------------------------------------------
C                  TEST OVERFLOW SWITCH. IF ON
C                  RETURN NEGATIVE VALUE OF I IN JC(1)
C                  ---------------------------------------------------------
        IF(ABS(A(K,J)).GT.10**75) IFL=1
        IF(IFL.EQ.1) GO TO 150
90      CONTINUE
91      CONTINUE
C                  ---------------------------------------------------------
C                  AX=B AND DET.(A) ARE NOW COMPUTED
C                  ---------------------------------------------------------
        GO TO (95,140),KI
C                  ---------------------------------------------------------
C                  PERMUTATION OF THE COLUMNS FOR MATRIX INVERSION
C                  ---------------------------------------------------------
95      DO 130  J=1,N
        IF (JC(J).EQ.J)  GO TO 130
        JJ=J+1
        DO 100  I=JJ,N
        IF  (JC(I).EQ.J)  GO TO 110
100     CONTINUE
110     JC(I)=JC(J)
        DO 120  K=1,N
        X=A(K,I)
        A(K,I)=A(K,J)
120     A(K,J)=X
130     CONTINUE
140       JC(1)=N
        IF(KD.EQ.1) V(1)=S
        RETURN
150     JC(1)=1-I
        IF(KD.EQ.1) V(1)=S
        RETURN 6
        END
**

@RESUME,E
```

VOLUME IV

PROBABILISTIC ANALYSIS OF RIGID AIRFIELD DESIGN
BY ELASTIC LAYERED THEORY

TABLE OF CONTENTS

* Copyrighted program. Information on the program can be obtained from the authors of this report.

LIST OF FIGURES

LIST OF TABLES

Chapter 1

INTRODUCTION

General

The design of military rigid (plain and reinforced) pavements is currently based upon the classical Westergaard free edge stress slab theory. It is this method that has historically been the basis for design methods found in present US Army Technical Manuals and U.S. Air Force Manuals. In addition to this approach, recent research conducted by Parker et al. (1) at the USACE Waterways Experiment Station has demonstrated that an equivalent, if not superior, design methodology based upon multi-layer linear elastic theory is also another feasible design approach for rigid pavements.

At present, both of these design methods are deterministic in that a unique pavement system is designed for the specific set of input variables necessary to solve the problem. In a deterministic model, all of these input variables are also unique in terms of their input magnitude.

Study Objective

The objective of the research study reported in this volume was to include the design parameter variability into the USACE design procedure based upon multi-layer elastic theory. This probabilistic analysis is based upon the design procedure developed by Parker et al. (1). The probabilistic methodology for the Westergaard based design method is presented in Volume III of this overall research study. The mathematical and analytical development of both probabilistic approaches used in this Volume and Volume III (Westergaard) are presented in Volume II of the research study.

Report Organization

The method of development used to establish the probabilistic multi-layer elastic theory design approach necessitated the investigation of determining an elastic theory based "Composite Modulus" for a multi-layered foundation system (i.e. a subbase layer over an existing subgrade soil). Because, this in itself, required a significant effort and investigation, this volume is subdivided into two major chapters. They are:

Chapter 2: Composite Modulus

Chapter 3: Probabilistic Analysis of PCC Airfield Design

In essence, Chapter 2 presents the solution to the "composite modulus" problem, which is an integral part of the overall probabilistic analysis presented in Chapter 3. The "composite modulus" has been evaluated from the BISAR multi-layered elastic theory computer code. As will be noted, this parameter is expressed not only in terms of the layer thickness and material properties but the specific loading conditions (aircraft) as well.

Chapter 3 dealing with the probabilistic analysis presents the development of the regression equation used for maximum stress computations used to evaluate the variability of the pavement design. Two solution approaches have been developed: (a) the approximate closed - form using first order Taylor series expansion and (b) Monte Carlo simulation. A computer program has been developed to solve this probabilistic approach and run example problems, a User's Guide and program listing are presented in Appendices.

Chapter 2

COMPOSITE MODULUS

Introduction

The concept of a composite modulus (of elasticity) was introduced to take into account the combined effect of a layered (subbase/subgrade) system underneath the rigid pavement on its performance. This parameter is commonly used in pavement design and evaluation and several methods were developed for computing the composite modulus as a function of layer thicknesses and moduli. It should be noted that the composite modulus, like the Equivalent Single Wheel Load, is defined as an equivalent modulus that will ultimately lead to the same correct response as with the original system of layers. Therefore, like the ESWL parameter, the composite modulus may be expected to depend upon: (1) the pavement response (maximum stress, strain or deflection) chosen for equivalency computations; (2) the load configuration and (3) the pavement geometry (layer thickness and moduli). It is stressed that all variables should be initially evaluated. Deleting anyone variable from the relation can be made only if its effect is found to be negligible. This chapter presents the evaluation of the composite modulus of elasticity for the case of a layered system. It is subdivided into the following sections:

(1) A brief description and discussion of the method for deriving the composite subgrade modulus of reaction (Westergaard model) in the current rigid pavement design system;

(2) A study of the effect of the composite modulus on rigid pavement performance in order to assess the degree of accuracy needed in the composite modulus evaluation;

(3) A comprehensive study of the effect of all design variables on the composite modulus of elasticity. All variables of the pavement geometry are included in a regression equation relating the composite modulus for one single wheel load to the pavement layer thicknesses and moduli. The load configuration effect is included in a separate equation relating the composite modulus for the given gear configuration to the composite modulus for one single wheel load.

Although, the composite modulus is not mandatory for the elastic layered systems (it is possible to compute the stress for the original system), it is of practical and economic interest to use the composite modulus in order to assess the value of additional layers and to simplify the design procedure.

Current U.S. Corp of Engineers Method for Determining the Composite Subgrade Modulus of Reaction (k_c Value)

In the current U.S.A.C.E. method (2) separate diagrams were developed for different materials. Figures 2.1 and 2.2 relate the composite subgrade modulus of reaction (denoted k_c) for specific unbound granular materials (well-graded crushed material and natural sand and gravel ($PI < 8$), respectively) to the base layer thickness and subgrade modulus of reaction. In Figure 2.3, the relationship is given for a broad range of subbase materials which are characterized through their modulus of elasticity. As explained in Volume III, the partially bonded rigid overlay pavement design equation for h_{doc} could also have been used.

Results of Figures 2.1 and 2.2 are based on field plate tests, while those of Figure 2.3 are based on computations using the elastic layered theory and correction procedures (3). In both diagrams, the equivalency criterion is deflection or stiffness, i.e., the composite

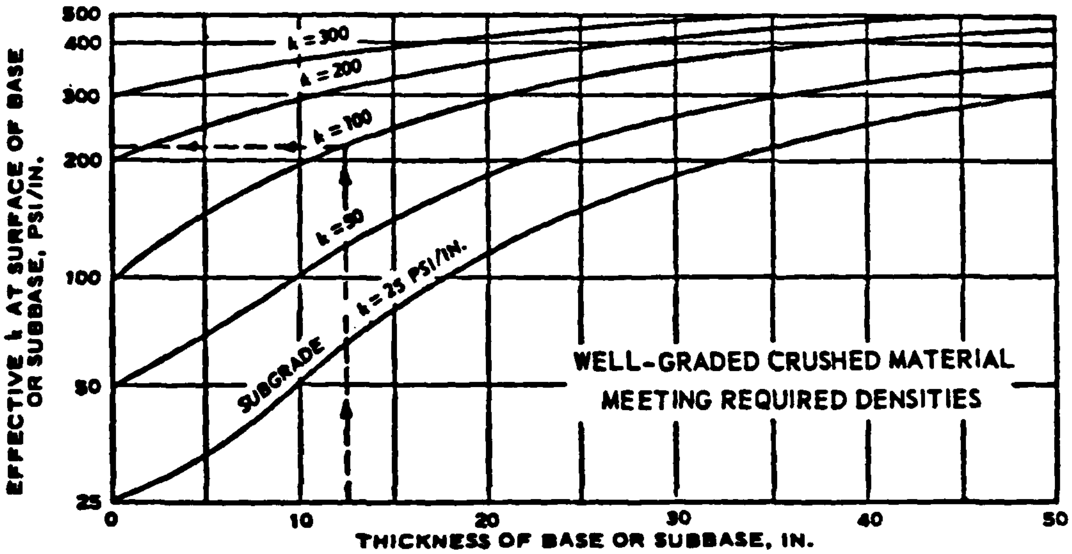

Figure 2.1 COMPOSITE SUBGRADE MODULUS OF REACTION FOR WELL-GRADED CRUSHED MATERIAL

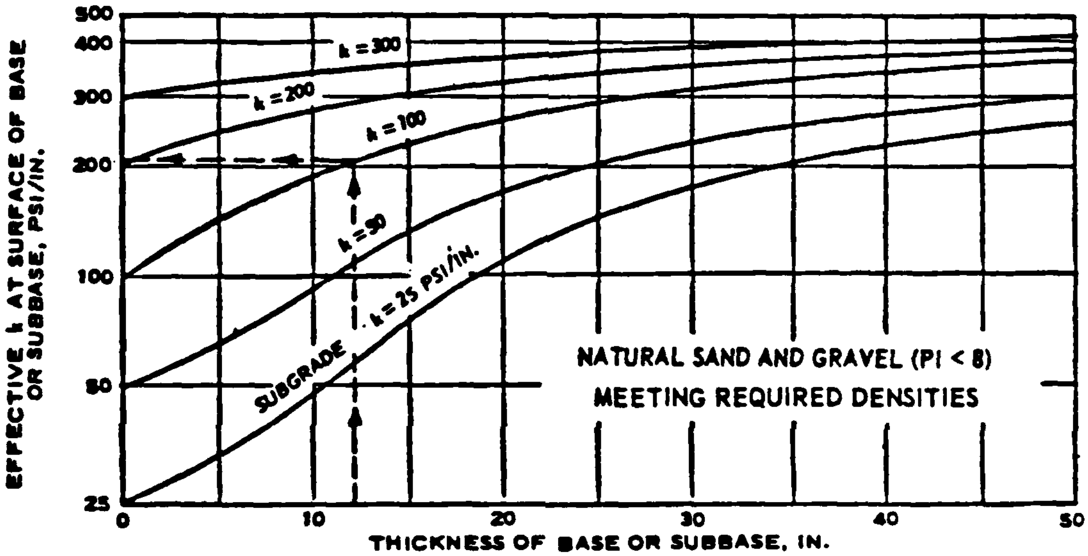

Figure 2.2 COMPOSITE SUBGRADE MODULUS OF REACTION FOR NATURAL SAND AND GRAVEL

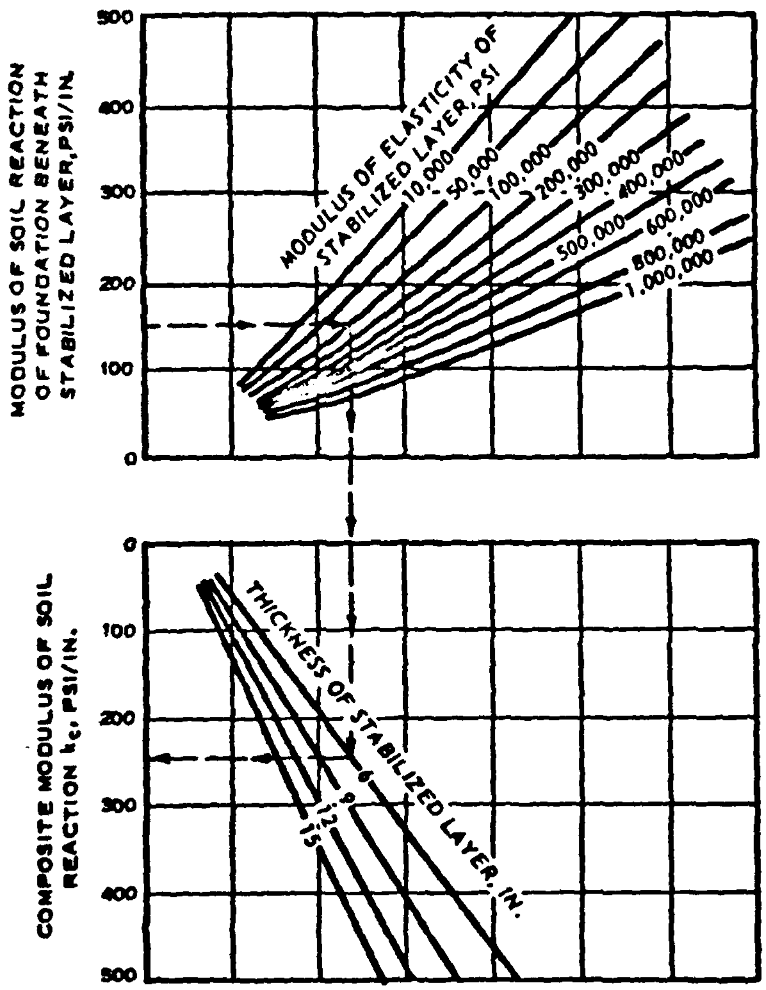

Figure 2.3 COMPOSITE SUBGRADE MODULUS OF REACTION FOR DIFFERENT BASE-SUBBASE MATERIALS

modulus of subgrade corresponds to equal stiffnesses of the equivalent subgrade and that of the original base-subgrade system. Therefore, the composite modulus computation scheme does not include the effect of slab thickness and modulus, neither that of the load configuration.

It should be remembered that the conventional military rigid pavement design method is based on the Westergaard theory where the subgrade is represented by either a dense liquid or a spring (i.e. two layer slab system). There is no possibility of taking care of the multilayer system underneath the pavement slab. Furthermore, the design method is based on limiting the maximum tensile stress at the bottom of the slab edge. It is not obvious whether the composite subgrade modulus evaluated using the above procedure will also result in equal maximum tensile stresses. In order to determine the composite subgrade modulus for equal maximum stresses, it would be necessary to conduct field tests on slabs, with stress or deflection bowl measurements. Such an approach would not be practical. However, with elastic multi-layered theory (unlike the Westergaard theory), it is possible to directly determine an equivalent or composite modulus, with any equivalency criterion parameter. In the ensuing analysis, the maximum tensile stress has been used as the equivalency criterion for obvious reasons.

Study of the effect of Composite Modulus on Rigid Pavement Performance

An alternative design procedure, using multi-layered elastic systems and center loading, to the existing one using Westergaard theory and edge loading has been developed by the U.S.A.C.E. (1). The salient features of this design analysis include:

(1) Computation of the maximum tensile stress (σ) at the bottom of the concrete slab, using elastic layered system theory. In these computations, it was assumed that (a) the interface between the concrete slab and the base or subgrade layer is frictionless and (b) the subgrade depth is finite and equals 20 ft; (c) a rigid (stiff) semi-infinite layer lies below the subgrade.

(2) Computation of the design factor (DF) is defined as:

$$DF = MR/\sigma \tag{2.1}$$

where

MR = modulus of rupture of concrete determined at age of 90 days.

(3) Computation of the allowable number of coverages (N) is obtained from the following relationships:

$$DF = 0.58901 + 0.35486 \log_{10} (N) \tag{2.2}$$

or

$$\ell n\ N = 6.4887\ MR/\sigma - 3.82192 \tag{2.3}$$

The data and the correlation of DF and N are shown in Figure 2.4.

The above design method will be used to study the effect of overestimating or underestimating the composite modulus on pavement performance. This can be expressed in terms of variation of N (the number of coverages) as a function of the variation of E_{comp} (the composite modulus). Substituting the function of σ (a function of the slab thickness and modulus, the composite or subgrade modulus and the loading conditions) into Equation 2.3 leads to the relationship between N and E_{comp}.

Expressing σ in its general form as:

$$\sigma = \sigma(h_1\ ,\ E_1/E_{comp},\ LOAD) \tag{2.4}$$

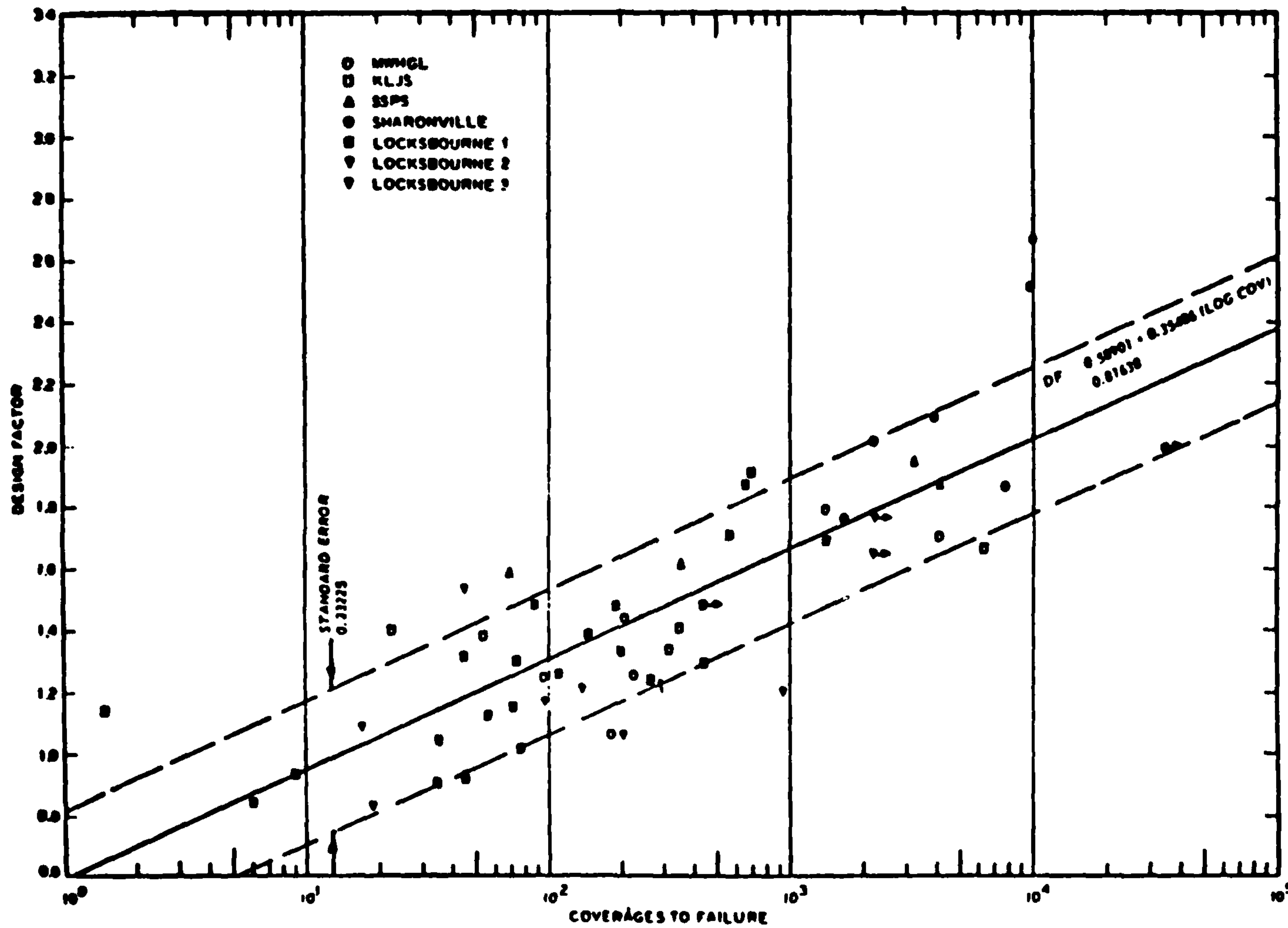

Figure 2.4 PERFORMANCE CRITERIA BASED ON DESIGN FACTOR

where

h_1 = slab thickness

E_1 = concrete modulus

E_{comp} = composite modulus subgrade modulus

LOAD = loading parameters

and combining equations 2.3 and 2.4 lead to:

$$\ell nN = 6.4887 \frac{MR}{\sigma(h_1,\ E_1/E_{comp},\ LOAD)} - 3.82192 \qquad (2.5)$$

It is seen that the effect of varying E_{comp} on N is not a simple function. It will vary with all pavement and load variables. Equation 2.5 is used to compute the deviation of N due to a deviation of E_{comp} in the following way:

(a) For a given aircraft, slab thickness, modulus and E_{comp}, compute N_o;

(b) Compute N_1 and N_2 at (0.8 E_{comp}) and (1.2 E_{comp}) corresponding to 20% deviation is E_{comp};

(c) Compute the relative deviation $100\ (N_2 - N_1) \div (\frac{N_o}{2})$;

(d) Repeat computations (b) and (c) for different percentage 15, 10 and 5% of E_{comp} deviations;

(e) Repeat computations for different sets of pavement variables.

(f) Repeat computations for different aircrafts.

Results of this analysis are given in Tables 2.1.a to 2.1.e for 5 different aircrafts AG-2, AG-9, AG-10, AG-12 and AG-13 where E_1 and MR are assumed equal to 4,000,000 and 700 psi respectively. It can be seen that the relative variation of N for N values between 100 and 100000 is 1.25 to 5.0

times that of E_{comp}. The magnifying factor increases with increasing N (corresponding also to increasing slab thickness and/or to increasing E_{comp}). It is larger for the heavier aircrafts (AG-9 to 13) than for the lighter aircraft (AG-2). Therefore, it is concluded that the design procedure is very sensitive to E_{comp} and more attention should be given to the determination of E_{comp}. Care should be taken in the use of E_{comp} since there is no maximum value of E_{comp} which may be used as there is with the composite modulus of subgrade reaction (k_{max} = 500 pci). It should be noted, however, that h is still the major governing factor. For example, if the computation of N is kept within 50% accuracy, determination of E_{comp} should be limited within less than 40% accuracy for the light aircraft and within less than 10% accuracy for the heavier aircrafts. The above conclusion must be taken into account in the design procedure and any applications of the procedure as discussed in the following paragraphs.

TABLE 2.1a: Effect of Composite Modulus Deviation on Number of Coverages for AG-2

Slab Thickness in.	Composite or Subgrade Modulus, psi	Number of Coverages-N	Relative Deviation of N for Different Relative Derivations of E_{comp} %			
			20	15	10	5
6	50,000	25	31	23	16	8
8	50,000	980	42	31	21	10
10	50,000	82,620	55	41	27	14
8	20,000	190	31	23	15	8
10	20,000	9500	41	31	20	10
12	20,000	1,070,000	54	40	27	13
8	10,000	73	25	18	12	6
10	10,000	2,650	33	25	16	8
12	10,000	203,000	43	32	21	11
8	5,000	34	19	14	10	5
10	5,000	967	25	19	13	6
12	5,000	54,950	32	24	16	8

TABLE 2.1b: Effect of Composite Modulus Deviation on Number of Coverages for AG-9

Slab Thickness, in.	Compute or Subgrade Modulus, psi	Number of Coverage-N	Relative Deivation of N for Different Relative Deviations of E_{comp} %			
			20	15	10	5
8	50,000	200	64	48	32	16
10	50,000	2,100	84	61	40	20
12	50,000	2,450	105	76	49	24
14	50,000	322,000	130	91	58	28
8	20,000	16	47	35	23	11
10	20,000	95	57	42	28	14
12	20,000	630	67	49	33	16
14	20,000	4,700	78	57	37	19
16	20,000	41,000	90	65	42	21
18	20,000	427,000	102	73	47	23
10	10,000	18	41	30	20	10
12	10,000	92	48	36	24	12
14	10,000	520	56	41	27	14
16	10,000	3,500	63	46	31	15
18	10,000	28,200	69	51	33	17
20	10,000	292,000	74	54	36	18
12	5,000	22	37	28	18	9
14	5,000	100	42	32	21	11
16	5,000	560	47	35	23	12
18	5,000	3930	50	37	24	12
20	5,000	38,400	49	36	24	12

TABLE 2.1c: Effect of Composite Modulus Deviation on Number of Coverages for AG-10

Slab Thickness, in	Composite or Subgrade Modulus, psi	Number of Coverages N	Relative Deviation of N for Different Relative Deviations of E_{comp}, %			
			20	15	10	5
8	50,000	5,000	89	65	43	21
10	50,000	102,500	119	85	55	27
8	20,000	170	64	48	31	16
10	20,000	1,580	80	59	39	19
12	20,000	16,400	99	71	46	23
14	20,000	188,000	120	85	55	27
8	10,000	26	48	36	24	12
10	10,000	157	60	45	29	15
12	10,000	1,040	73	54	35	18
14	10,000	7,470	87	63	41	20
16	10,000	59,200	101	73	47	23
18	10,000	520,000	116	82	53	26
10	5,000	26	46	35	23	11
12	5,000	121	55	41	27	13
14	5,000	630	63	46	31	15
16	5,000	3,800	68	50	33	16
18	5,000	26,600	70	51	34	17
20	5,000	236,000	66	48	31	15

TABLE 2.1d: Effect of Composite Modulus Deviation on Number of Coverages for AG-12

Slab Thickness, in.	Composite or Subgrade Modulus, psi	Number of Coverages N	Relative Deviation of N for Different Relative Deviations of E_{comp}, %			
			20	15	10	5
10	50,000	1,350	71	53	35	17
12	50,000	16,800	92	67	44	22
14	50,000	243,000	117	84	54	26
10	20,000	82	53	39	26	13
12	20,000	550	65	48	32	16
14	20,000	4,060	79	58	38	19
16	20,000	33,000	94	68	45	22
18	20,000	294,000	112	80	52	25
10	10,000	17	40	30	20	10
12	10,000	80	50	37	24	12
14	10,000	410	60	44	29	15
16	10,000	2,300	71	52	34	17
18	10,000	13,600	83	61	40	20
20	10,000	87,000	96	70	45	22
12	5,000	18	39	29	20	10
14	5,000	68	47	35	23	12
16	5,000	276	55	41	27	14
18	5,000	1,200	64	47	31	16
20	5,000	5,700	72	53	35	17
22	5,000	29,200	79	58	38	19
24	5,000	166,000	85	62	40	20

TABLE 2.1e: *Effect of Composite Modulus Deviation on Number of Coverages for AG-13*

Slab Thickness in.	Composite or Subgrade Modulus, psi	Number of Coverages N	Relative Deviation of N for Different Relative Deviation of E_{comp}, %			
			20	15	10	5
11	50,000	52	52	39	26	13
13	50,000	277	62	46	30	15
15	50,000	1,550	72	53	35	17
17	50,000	9,400	82	60	39	20
19	50,000	63,300	94	68	44	22
13	20,000	28	41	31	21	10
15	20,000	112	48	35	23	12
17	20,000	488	54	40	26	13
19	20,000	2,330	60	45	30	15
21	20,000	12,500	68	50	33	16
23	20,000	77,700	76	55	36	18
15	10,000	27	35	26	18	9
17	10,000	100	40	30	20	10
19	10,000	400	45	33	22	11
21	10,000	1,780	50	37	24	12
23	10,000	9,080	55	40	27	13
25	10,000	55,300	60	44	29	15
17	5,000	30	31	23	16	8
19	5,000	103	35	26	17	9
21	5,000	402	38	28	19	9
23	5,000	1,800	41	31	20	10
25	5,000	9,750	43	32	22	11

Determination of the Composite Modulus

The composite modulus is determined on the basis of equal maximum tensile stress, i.e. the same maximum tensile stress will occur when analyzing either the original system composed of a concrete layer on a base-subbase-subgrade system or an equivalent system composed of the same concrete layer on the top of the "equivalent" composite subgrade. Like the ESWL, all pavement variables (h_i, E_i) and the loading (gear) variables may be expected to affect the composite modulus. In the following,it was assumed that the E_{comp} function was of the following mathematical form.

$$E_{comp} = E_3 \, f_1^{\,f_2} \tag{2.6}$$

where

f_1 = a function of the pavement geometry only, under the application of one wheel load only

f_2 = a function of the loading (gear) conditions only

Composite Modulus for One Wheel Load

The following range of variables were included in the computation of the maximum tensile stress and computation of the E_{comp} for one wheel load.

(a) Wheel load of AG-2, AG-4, AG-6, AG-9, AG-10, AG-12 and AG-13;

(b) h_1 - thickness of concrete layer varying from 6 in. to 25 in.; for each aircraft group, four thicknesses covering the whole range of expected design thicknesses (see Table 2.2);

(c) h_2 - thickness of base-subbase layer of 6 and 12 in;

(d) E_2 - modulus of base-subbase layer of 50,000, 200,000 and 800,000 psi to include stabilized material, as well as unbound materials;

(e) E_3 - modulus of subgrade of 4000, 12,000 and 35,000 psi.

The form of the function f_1 was chosen to include the limiting case of $h_2=0$ and $E_2=E_3$ where no base-subbase layer exists. Several functions of the pavement geometry variables were assumed. The following equation gave the most satisfactory correlation coefficient and standard error of estimate:

$$\ell n(E_{comp}/E_3)_1 = a_1 \ell n\frac{E_2}{E_3} +$$

$$\ell n\alpha \{a_2 \ell n h_1 + a_3 \ell n\frac{E_1}{E_3} + a_4 \ell n(h_2+1) + a_5 \ell n\frac{E_2}{E_3}$$

$$+ a_9(\ell n h_1)^2 + a_{10} \ell n h_1 \ell n\frac{E_1}{E_3} + a_{11}(\ell n\frac{E_1}{E_3})^2 \} +$$

$$(\ell n\alpha)^2 \{a_6 \ell n h_1 + a_7 \ell n(h_2 + 1) + a_8 \ell n\frac{E_2}{E_3}\} \tag{2.7}$$

where

$$\alpha = \sqrt[3]{\frac{E_2}{E_3}} \cdot (h_2 + 1)$$

$(E_{comp}/E_3)_1$ = the ratio between E_{comp} and E_3 for one wheel load

$a_1 \ldots a_{11}$ - regression coefficients

$a_1 = 0.434783$, $a_2 = -0.134317$, $a_3 = 0.0816432$,

$a_4 = -0.0807450$, $a_5 = -0.190090$, $a_6 = -0.164512$,

$a_7 = 0.119533$, $a_8 = 0.0550536$, $a_9 = 0.0943027$,

$a_{10} = 0.0102537$, $a_{11} = -0.0118395$

The R^2 of the multiple correlation was 0.994 and the standard error of estimate 0.0776 corresponded to a standard deviation of 8.0% in the E_{comp} evaluation. It should be noted that the equation has a zero intercept and

$\ell n\alpha$, $\ell n\frac{E_2}{E_3}$ equal 0 when $h_2 = 0$ and $E_2 = E_3$.

A simpler equation, with 5 terms only was also derived:

$$\ell n(f_1/E_3) = (\ell n\alpha)^2 \{a_1' \ell nh_1 + a_2' \ell n(h_2+1) + a_3' \ell n\frac{E_2}{E_3}\} + \ell n\alpha \{a_4'(\ell nh_1)^2 + a_5'(\ell n\frac{E_1}{E_3})^2\} \qquad (2.8)$$

where

$a_1' = -0.161317$, $a_2' = 0.0977117$, $a_3' = 0.0358696$
$a_4' = 0.0802967$, $a_5' = -0.00196301$

with $R^2 = 0.991$ and standard error of estimate = 0.095 corresponding to a standard deviation of 10% in the E_{comp} evaluation. It should be noted that (1) the contribution of the terms with h_1 and E_1/E_3 to the regression was relatively substantial, partly due to the wide range of h_1 and E_1/E_3 included in the analysis; (2) the general trend of the effect of pavement geometry on the E_{comp} parameter was as follows: (a) E_{comp}/E_3 increases as h_2 and/or E_2/E_3 increases; (b) E_{comp}/E_3 decreases as h_1 increases and/or E_1/E_3 decreases; (3) within the range of the wheel loads used, it seems that the relationship is independent of the radius of the contact area; (4) for the 378 data points used in the regression the deviation in evaluating E_{comp} with the regression equation exceeded 25% in some but few cases. Therefore, depending on the accuracy level required for predicting the number of coverages, equation 2.7 may or may not be adequate for design purposes. According to the above analysis of the effect of composite modulus deviation on number of coverages, a deviation of 10 percent in E_{comp} evaluation resulted in about 25 and 40 percent deviation in the expected number of coverages for the light and heavy loads respectively. A procedure for improving the composite modulus evaluation was developed,

based on only one run of the BISAR program. The procedure, presented in Chapter 3 must be used at least for all heavy aircrafts; (5) equation 2.8 predicts the dependence of E_{comp} upon the pavement geometry variables quite well. Because it is easier to differentiate, this equation was used in the probabilistic approach, in the evaluation of the variance.

Composite Modulus for Different Aircrafts

The effect of loading (gear) conditions, that is, the number of wheels and wheel configuration is expressed in Equation 2.6 by the f_2-function. For one wheel load, $f_2 = 1$ (i.e. for AG-1, 2 and 3). The loading conditions seem to have a similar effect on E_{comp} as h_1, the concrete layer thickness. From previous paragraphs, it was stated that while all other variables are kept constant, increasing the h_1 resulted in a decrease of the E_{comp} term. This can be interpretated as follow: increasing h_1 increases the pavement stiffness, the radius of relative stiffness (ℓ) and the size of the deflection bowl; The relative base-subbase layer thickness h_2/ℓ decreases, entraining a reduction of its effect on E_{comp}. The same relation between E_{comp} and number of wheels exists: when the number of wheel loads increases, the deflection bowl size increases, and the relative base-subbase layer thickness decreases. Therefore, the f_2-function may logically be expected to decrease as the number of wheels increases. Instead of expressing f_2 for each aircraft group, it was decided to simplify the analysis of f_2 and confine it to three types of wheel loading configuration, (namely two, four and six gear wheels). Figures 2.5 to 2.9 show the relationship between the E_{comp}/E_3 parameter for one wheel load and E_{comp}/E_3 for AG-4, 6, 9, 13 and 10. A simple log-log function without intercept was fitted to the computation results for the two, four and six wheel assemblies, (AG-4 & 6, 9 & 13 and 10 respectively):

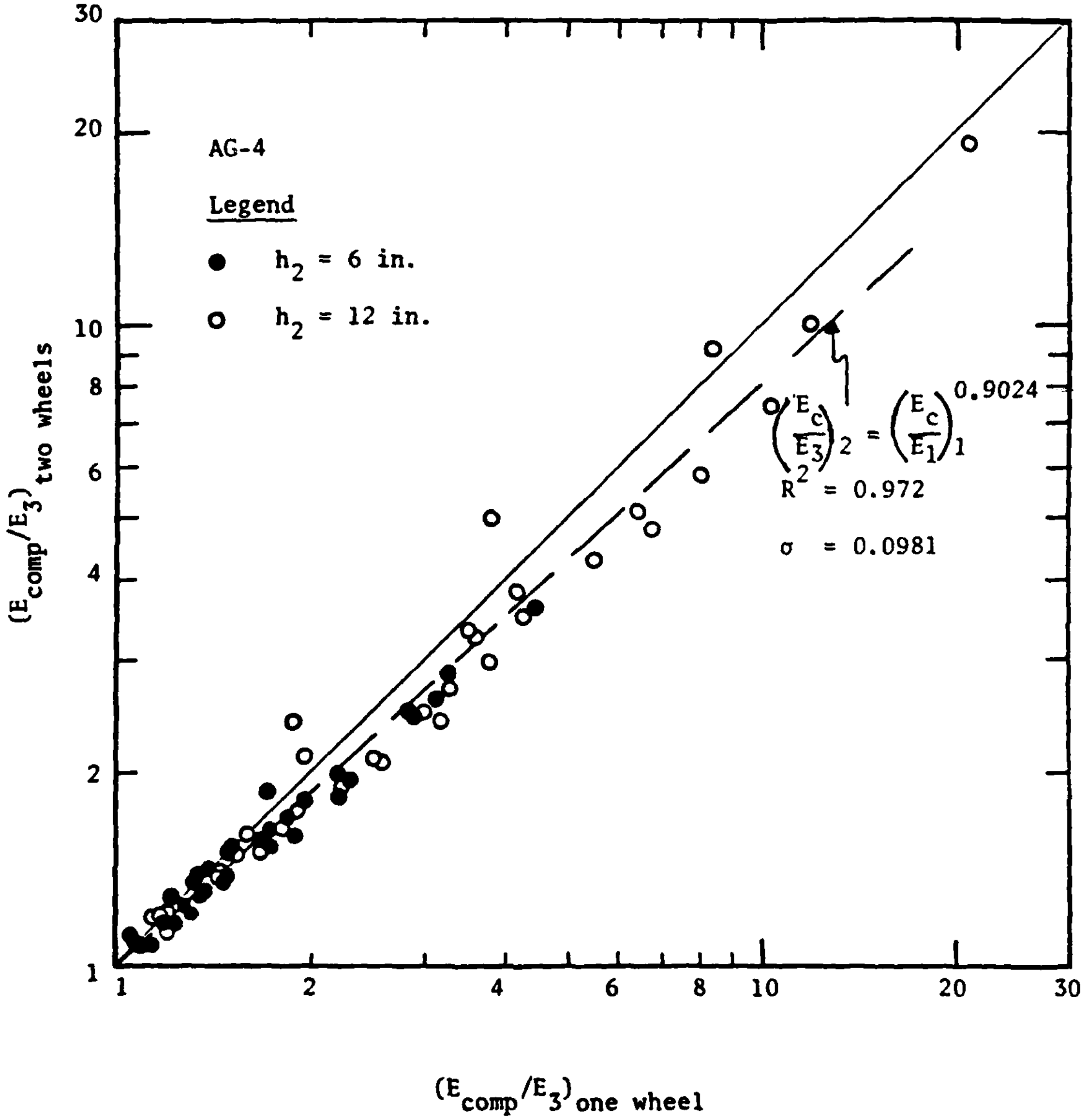

Figure 2.5 THE f_2 - FUNCTION FOR AG-4

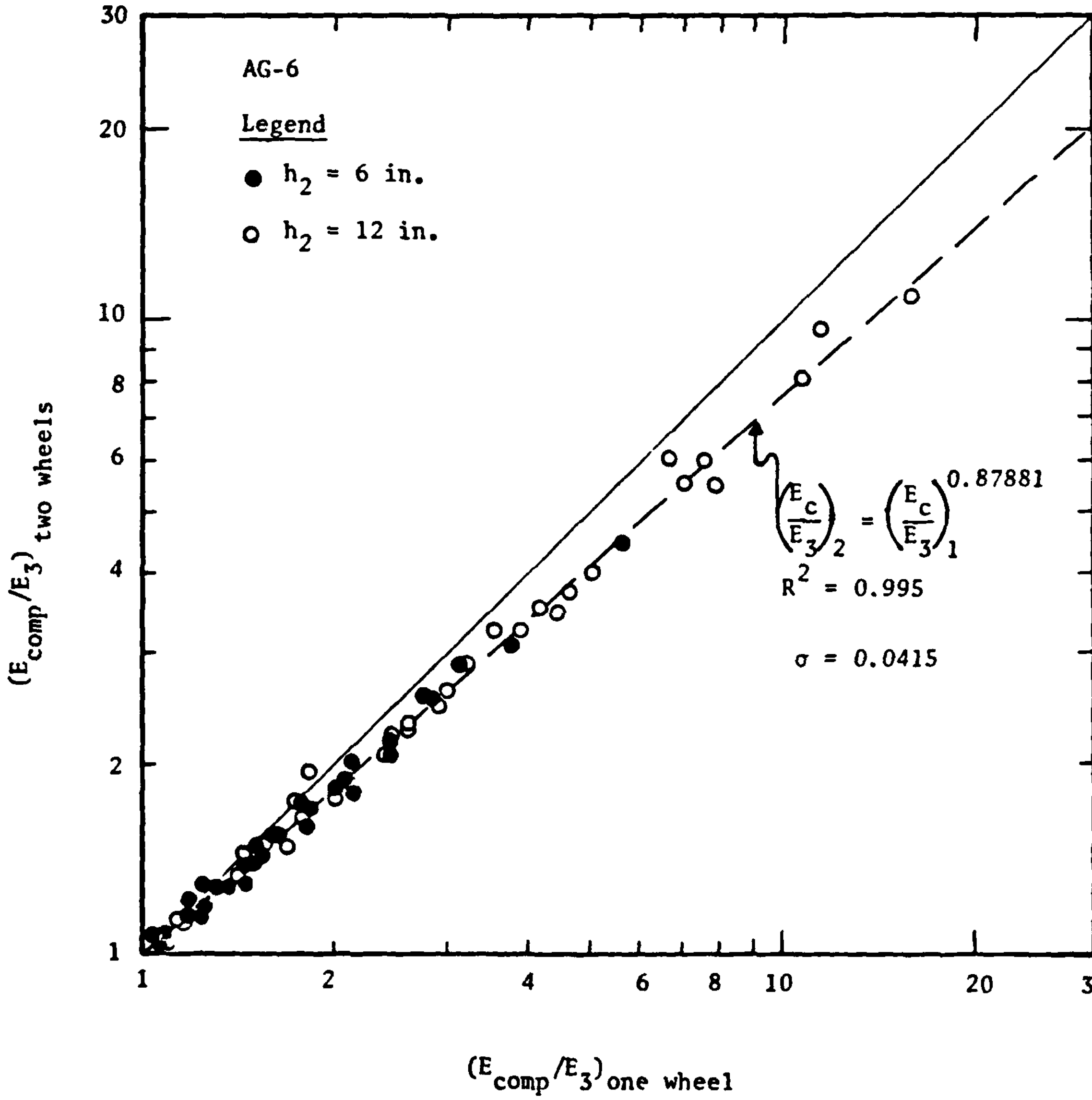

Figure 2.6 THE f_2 - FUNCTION FOR AG-6

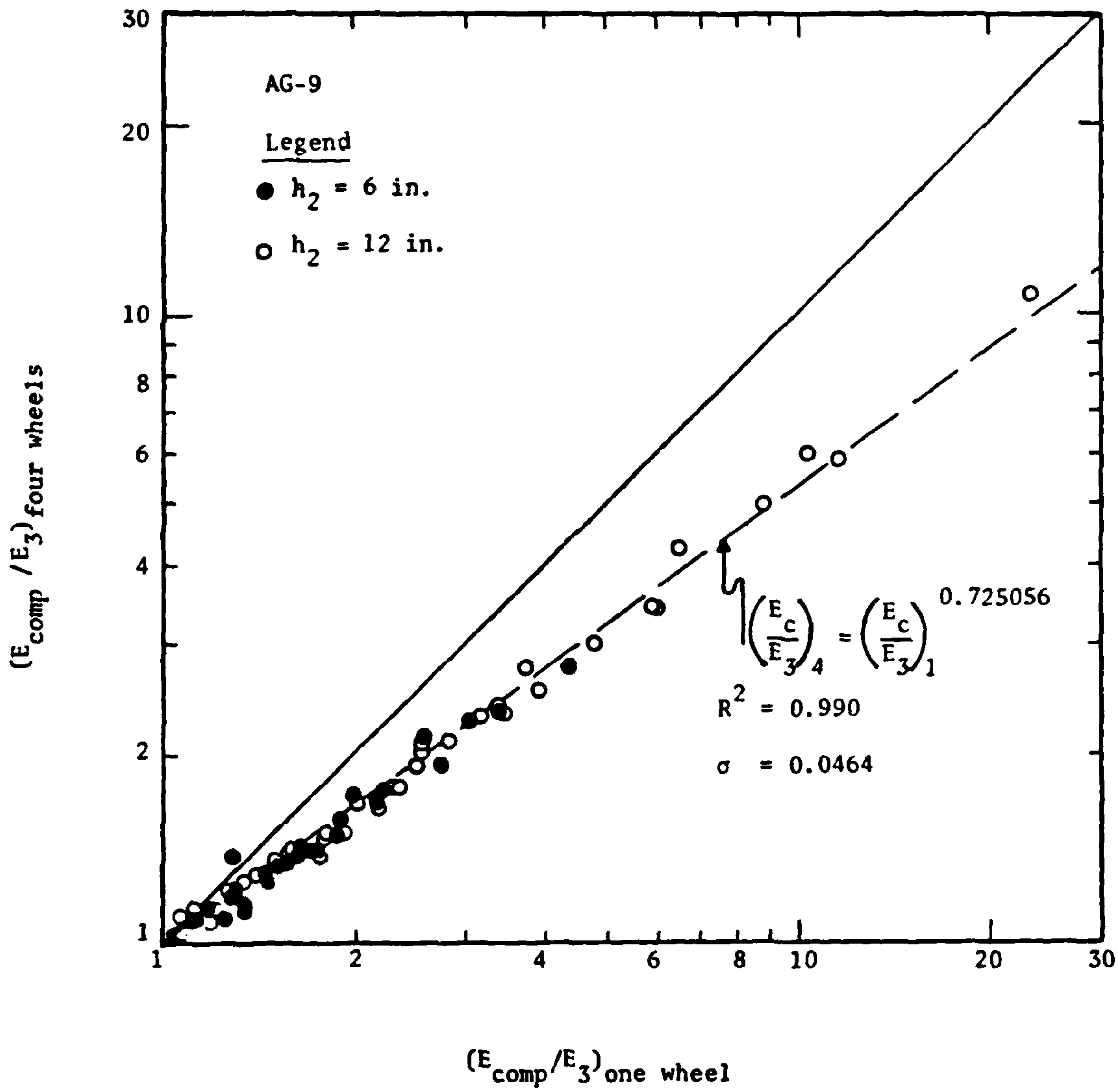

Figure 2.7 THE f_2 - FUNCTION FOR AG-9

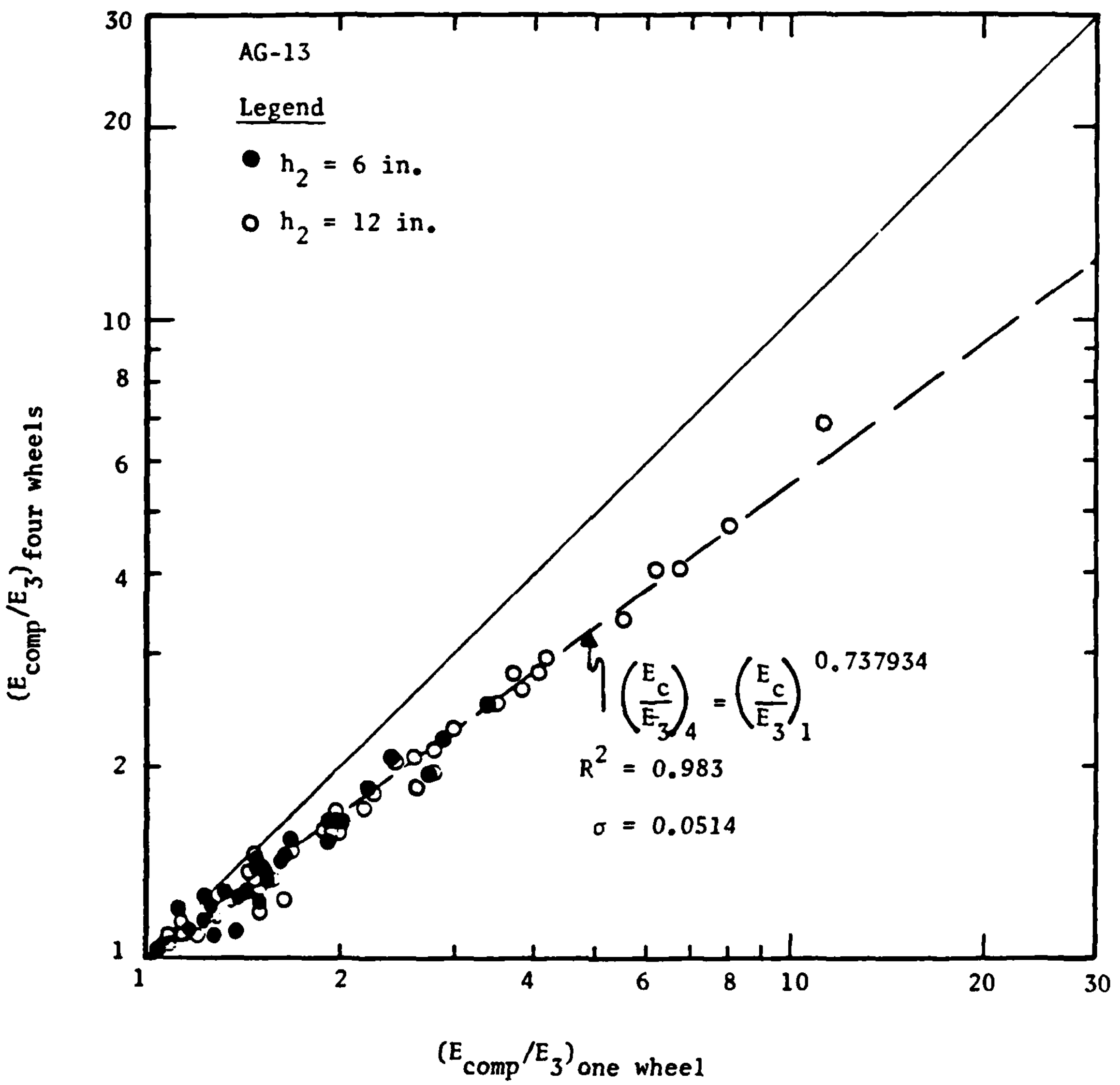

Figure 2.8 THE f_2 - FUNCTION FOR AG-13

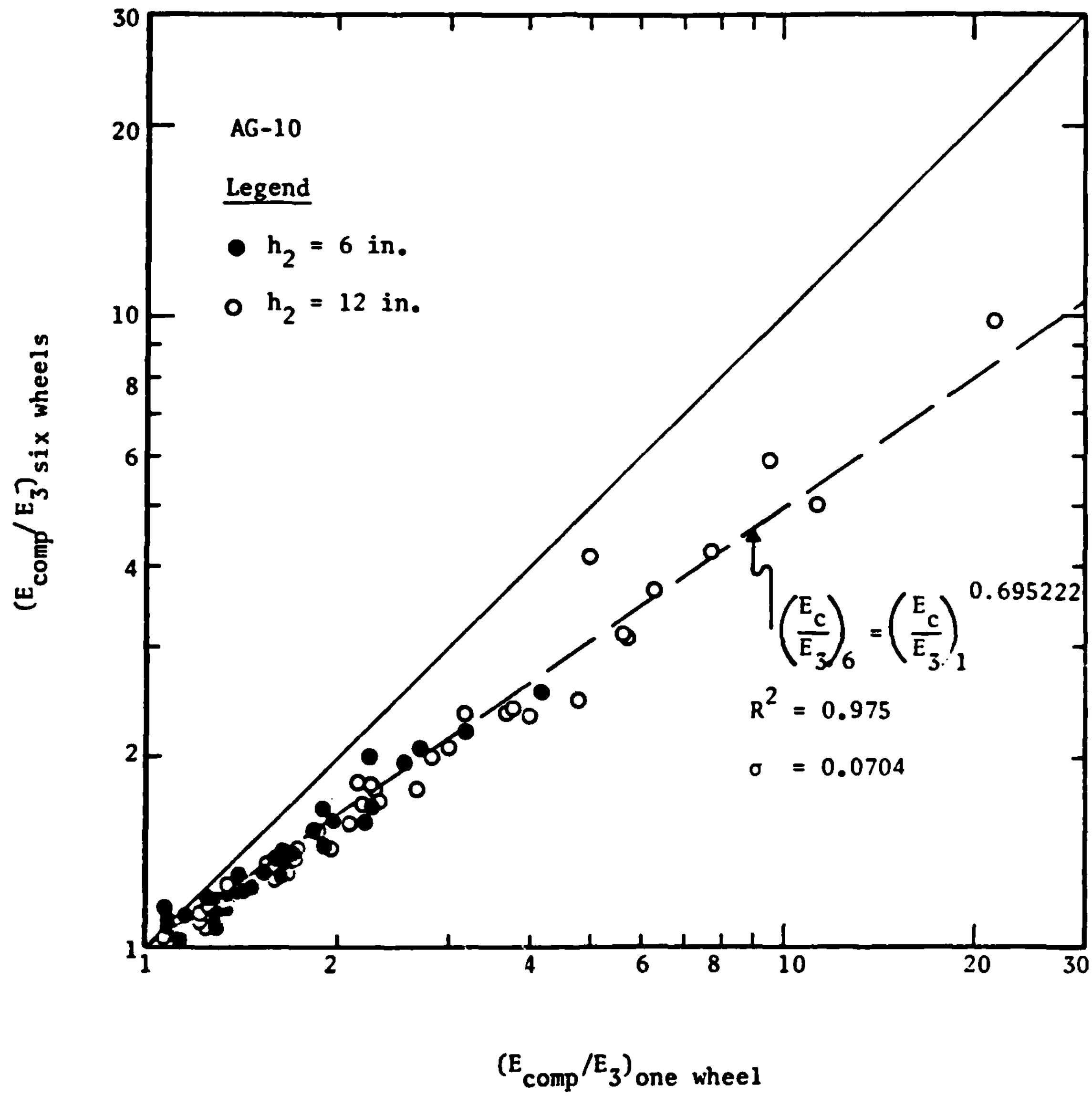

Figure 2.9 THE f_2 - FUNCTION FOR AG-10

$$\left(\frac{E_c}{E_3}\right)_n = \left(\frac{E_c}{E_3}\right)_1^{a_n}$$

$$\ell n(E_{comp}/E_3)_n = a_n \ell n(E_{comp}/E_3)_1 \qquad (2.9)$$

where

$(E_{comp}/E_3)_n$ = the E_{comp}/E_3 for n-wheels

a_n = regression coefficients, for n-wheels

$a_1 = 1.0$, $a_2 = 0.889813$, $a_4 = 0.730507$, $a_6 = 0.695222$

The R^2-coefficients were 0.984, 0.987, 0.975 and the standard errors of estimate were 0.073, 0.0493, 0.0704 for 2, 4 and 6 wheels respectively. It is noted that (1) the composite modulus is strongly affected by the loading conditions, reaching about half the value of one wheel for the 4 and 6-wheel loading conditions; (2) the relationship f_2 could be improved if some pavement geometry variables were included in equation 2.9: The a_n-regression coefficient seems to increase as h_1 increases. However, it is believed that equation 2.9 is quite accurate and there is no need for expanding it.

Evaluation of the Results

The results of the above computations are compared with those of the current USACE procedure for determining the composite subgrade modulus of reaction. A summary of the results is presented in Table 2.3 where: (1) k_{sg}- the subgrade modulus of reaction is computed according to Parker et al. (1), using:

$$\log k_{sg} = (\log E_3 - 1.415)/1.284 \qquad (2.10)$$

(2) k_{comp} - the composite subgrade modulus of reaction is evaluated from Figure 2.3; (3) $k(E_{comp})$ is evaluated using equation 2.10 and substituting E_{comp} for E_3; (4) the range of $k(E_{comp})$ includes all aircrafts and concrete layer thicknesses given in Table 2.2. It should be noted that the upper limit corresponds to the light aircraft (AG-2) and very thin concrete layer (6") while

TABLE 2.2: Summary of Concrete Layer Thickness Used in the Analysis of Composite Modulus for One Wheel

Aircraft Group	Concrete Layer Thickness, in.
2	6
4	7,10,12,15
6	7,10,13,17
9	8,12,15,19
10	8,12,15,20
12	10,15,19,24
13	11,16,20,25

TABLE 2.3: Comparison of Composite Subgrade Moduli of Reaction

h_2,in	E_2,psi	E_3,psi	$k_{sg}(E_3)$,pci	k_{comp},pci	$k(E_{comp})$,pci
6	50,000	35,000	275	360	278-295
		12,000	120	160	124-154
		4,000	50	50	53- 75
	200,000	35,000	275	490	307-389
		12,000	120	220	134-207
		4,000	50	60	65-113
	800,000	35,000	275	(970)	315-584
		12,000	120	390	165-349
		4,000	50	120	78-226
12	50,000	35,000	275	(550)	282-317
		12,000	120	240	134-202
		4,000	50	65	63-120
	200,000	35,000	275	(720)	342-577
		12,000	120	340	174-397
		4,000	50	105	78-285
	800,000	35,000	275	(1270)	428-1352
		12,000	120	(580)	242-1125
		4,000	50	200	133-984

Note: Numbers in parentheses are obtained from extrapolation in Figure 2.3 . The value should be 500 pci

the lower limit corresponds to heavy aircrafts and thick concrete layer.

The values of k_{comp} for weak subgrade were evaluated by extrapolating the results of Figure 2.3. For the case of $E_3 = 4000$ psi ($k_{sg} = 50$ pci) and $E_2 = 50,000$ psi, higher k_{comp} values are obtained from Figure 2.2 than from extrapolating Figure 2.3.

Comparison of the k_{comp} and $k(E_{comp})$ in Table 2.3 shows that: (1) for weak subgrades, the $k(E_{comp})$ is either in the range of or higher than the k_{comp} evaluated with the current USACE procedure. However, if the extreme cases of very thin concrete layer were excluded from $k(E_{comp})$ and the k_{comp} were derived from Figure 2.2, the discrepancy would be quite substantially reduced; (2) for relatively strong subgrades, the $k(E_{comp})$ is lower than the k_{comp}, and exceeds 500 pci in few cases only. This result seems to support the current USACE procedure where the subgrade modulus of reaction (or the composite) is limited to a maximum value of 500 psi. It appears that the computation results of the composite modulus are in general agreement with the current USACE k_{max} value, ahthough more research studies are necessary to fully support this general observation. Furthermore, the present derivation takes into account all of the design variables which appear to have different effects on the pavement performance.

Summary and Conclusions

A composite modulus computation scheme has been presented that includes all pavement geometry and loading condition variables. It was found that the composite modulus of elasticity is not only a function of the base-subbase and subgrade layers, but also a function of the concrete layer and load configuration.

The study of the effect of E_{comp} on allowable number of coverages (or pavement performance) clearly showed that the degree of accuracy achieved in

the design (predicted number of coverages) is strongly related to that achieved in E_{comp} evaluation. The relative deviation in the allowable number of coverages is 1.2 to 5 times the relative deviation in E_{comp}.

These two results- of the function E_{comp} being dependent upon all variables and that of the sensitivity analysis, emphasize the importance of the evaluation of E_{comp}, and give quantitative relationships to determine the required degree of accuracy.

Chapter 3

PROBABILISTIC ANALYSIS OF PCC AIRFIELD DESIGN USING ELASTIC LAYERED THEORY

Introduction

The analysis presented in this chapter is based on the mathematical formulation given in Volume II, namely the approximate closed-form probabilistic approach and the probabilistic simulation approach. Both methods require stress computations which can be handled with computer programs such as the Shell BISAR code. However, while the use of a computer program is justified for the design of a particular pavement, it does become unrealistic and uneconomical for the probabilistic approach, due to the large computer time needed. The analysis was therefore confined to specific critical aircraft type included in the military (USAF) aircraft classification scheme (AG-1 to 13). Solutions for other aircraft type may be developed by following the procedure noted below and developing an equation for the maximum tensile stress.

The probabilistic approach presented in the following paragraphs includes:

(1) Stress computations and derivation of an equation for the maximum tensile stress for each of 13 aircraft types: USAF classification AG-1 to 13;

(2) Derivation of the relationships between variances of the dependent and independent variables for the approximate closed-form approach. The linear or first order Taylor series expansion is assumed as presented in Volume II.

(3) Formulation of a simulation model which bypasses the linear assumption in the closed-form approach.

(4) Run examples, to compare between the closed-form approach and the simulation one, at different variation levels of the design parameters.

Stress Computations

In the "normal" linear elastic layered system, the stress at any point is a function of the layer thicknesses (expressed in terms of the radius of contact area) and of the modular ratios of the layers and the subgrade. However, because of the assumptions made by Parker et al. (1) concerning the depth of the subgrade, the modulus of the rigid layer underneath the subgrade (chosen to be equal to 1,000,000 psi), and the value of the friction coefficient at the interface between the concrete and base on subgrade layers, it is not possible to use dimensionless variables. Therefore stress computation were conducted for all 13 aircrafts of the USAF classification scheme (AG-1 to 13) for the following ranges of variables (see Table 3.1):

(1) Modulus of concrete - E_1 = 4,000, 000 psi and Poisson's ratio of 0.2;

(2) Concrete layer thickness - h_1, choosen to cover a wide range of design possibilities. Four different values were assigned for each aircraft group, as shown in Table 3.1;

(3) Subgrade moduli or composite subgrade moduli of 4,000, 12,000, 35,000 and 100,000 psi and Poisson's ratio of 0.4.

(4) Subgrade thickness of 20 ft = 240 in.

(5) Elastic modulus of the stiff infinite layer beneath the subgrade of 1,000,000 psi.

(6) Friction coefficient (interface compliance) at the interface of the concrete and subgrade layer of 1,000.(defined in the BISAR program).

Table 3.1 Summary of Input Data for Stress Computations

AG No.	Radius of Contact area in.	Contact Pressure psi	Wheel Coordinates			Max.Stress Under Wheel No	Concrete Layer Thicknesses in.
			Wheel No	X	Y		
1	9.3560	98.1818	1	0.0	0.0	1	6 9 11 14
2	5.6419	270.0	1	0.0	0.0	1	6 9 11 14
3	8.7586	186.722	1	0.0	0.0	1	7 10 12 15
4	11.2838	87.1875	1	0.0	0.0	1	7 10 12 15
			2	60.0	0.0		
5	7.2471	155.4545	1	0.0	0.0	1	7 10 13 17
			2	26.0	0.0		
6	7.4422	156.9684	1	0.0	0.0	1	7 10 13 17
			2	30.5	0.0		
7	8.6856	190.4008	1	0.0	0.0	1	8 11 14 18
			2	34.0	0.0		
8	8.3302	183.0275	1	0.0	0.0	1	8 12 15 19
			2	34.5	0.0		
			3	0.0	56.0		
			4	34.5	56.0		
9	8.1369	186.6587	1	0.0	0.0	1	8 12 15 19
			2	32.5	0.0		
			3	0.0	48.0		
			4	32.5	48.0		
11	9.6738	180.3061	1	0.0	0.0	1	8 12 15 20
			2	54.0	0.0		
			3	0.0	64.0		
			4	54.0	64.0		
12	8.8310	188.5714	1	0.0	0.0	4	10 15 19 24
			2	44.0	0.0		
			3	0.0	58.0		
			4	44.0	58.0		
13	9.2189	233.7079	1	-37.0	0.0	2	11 16 20 25
			2	0.0	0.0		
			3	62.0	0.0		
			4	99.0	0.0		
10	9.5246	105.6728	1	34.0	0.0	2	8 12 15 20
			2	0.0	0.0		
			3	-53.0	0.0		
			4	-87.0	0.0		
			5	-2.50	65.0		
			6	-50.5	65.0		

The general form of the maximum tensile stress at the bottom of the concrete layer is given by:

$$\sigma = \sigma(h_1, E_1, E_{comp}, \text{loading conditions}) \quad (3.1)$$

where E_{comp} = the composite modulus of subgrade is equal to the subgrade modulus when no base-subbase layer exists.

The computation results for each aircraft were used to develop a regression equation for prediction the maximum tensile stress for any combination of pavement geometry variables. The following model fitted all thirteen aircrafts:

$$\sigma = a_o + a_H h + a_E E + a_{EH} Eh + a_{E2H} E^2 h + a_1 \frac{\ln\beta}{h} + a_2/\beta + a_3 \frac{\ln\beta}{\beta h} + a_4 \left(\frac{\ln\beta}{h}\right)^2 + a_5/\beta^2 \quad (3.2)$$

where

- $h = h_1$

- $E = E_1/E_{comp}$

- $\beta = h^3\sqrt{E}$

- h_1 = concrete layer thickness, inch.

- E_1 = modulus of concrete, psi.

- E_{comp} = composite or subgrade modulus, psi.

- σ = maximum tensile stress, psi.

- a_i = regression coefficients.

The values of the regression coefficient, R^2 and the relative error of estimate are given in Table 3.2. It is seen that the regression equation is an excellent one, giving a maximum standard error of 0.8%.

Table 3.2 Regression Coefficients, R^2 and Standard Error of Estimate for Stress Computations

AG	a_o	a_H	a_{H^2}	a_{EH}	a_{E2H}	a_1	a_2	a_3	a_4	a_5	R^2	σ-%
1	47.13551	-3.114693	-0.02416481	0.0009148971	0.	0.	1640.099	-10162.07	1849.660	7426.127	.99999	0.21
2	-29.09671	-0.8760680	-0.03832524	0.001772930	0.	0.	1456.306	-808.1055	2288.042	-21056.89	.99999	0.17
3	284.7195	-11.03868	-0.04843520	0.	0.0000017	-415.1382	0.	-12672.95	3433.536	20821.21	.99997	0.29
4	51.76199	-2.852370	0.06405583	-0.002479894	-0.0000010724	217.1736	0.	-20064.09	2127.983	72434.43	.99997	0.28
5	84.40505	-3.809808	0.	-0.00071885	0.	0.	628.4619	-18569.88	2751.819	55943.80	.99994	0.44
6	103.4132	-4.370129	0.	-0.00078082	0.	0.	0.	-19993.78	2785.334	75051.80	.99994	0.44
7	158.8361	-6.001916	0.	-0.001021292	0.	0.	0.	-35657.54	4331.092	137800.8	.99995	0.38
8	-49.08532	0.	0.1624640	-0.008325638	0.	949.1228	-5345.551	-41370.78	2769.408	310403.1	.99981	0.78
9	-66.52055	0.	0.1370117	-0.007595463	0.	1027.890	-3937.352	-52771.66	3098.378	342184.9	.99983	0.75
10	-167.6606	2.671640	0.2175972	-0.005508302	-0.0000029737	1103.715	-1893.149	-30801.11	1564.013	185562.8	.99985	0.67
11	-394.8007	7.124734	0.2578348	-0.006082276	-0.0000040731	1798.772	0.	-34739.42	1875.246	193755.1	.99980	0.80
12	-164.3000	2.081288	0.1425741	-0.002873140	-0.0000010815	1188.027	-2294.815	-46476.85	2808.230	298345.3	.99998	0.24
13	224.6386	-4.946335	0.07221752	-0.003123284	0.	0.	-5139.840	-75798.53	7112.995	460427.8	.99991	0.54

Although equation 3.2 contains quite a large number of terms in order to achieve the high degree of accuracy needed, it is quite easily programmable on micro-computers and calculators. It should be noted that: (a) the maximum tensile stress was found to be higher for a 6-wheel gear than for the 12-wheel gear of the C-5 (AG-10); (b) maximum tensile stress was found to correspond to a 4-wheel gear in some combinations of h and E and to an 8-wheel gear in other combinations for the B-747 (AG-12). Equation 3.2 expresses the maximum stress corresponding to either 4 or 8-wheel gear.

Equation 3.2 covers a quite wide range of values of the pavement design parameters and design analysis. It should be remembered that equation 3.2 applies for a two-layer system only. When the layered system underneath the concrete contains a base-subbase layer, it should be transformed into an equivalent subgrade and its composite modulus evaluated according to Chapter 2 of this volume.

The Approximate Closed Form Probabilistic Approach

In the approximate closed-form probabilistic approach, the average and the variance of the random variables are given by the following equations, assuming linear or first order TAylor series expansion of the variable function:

$$E[g(x_i)] = g(x_i)\big|_{\bar{x}_i} \tag{3.3a}$$

$$E[(g(x_i)-\mu)^2] = \mathrm{Var}[g(x_i)] = \sum_{i=1}^{n}\left(\frac{\partial g(x_i)}{\partial x_i}\bigg|_{\bar{x}_i}\right)^2 \mathrm{Var}[x_i] +$$

$$\sum_{i=1}^{n}\sum_{\substack{k=1\\k\neq i}}^{n}\left(\frac{\partial g(x_i)}{\partial x_i}\bigg|_{\bar{x}_i}\right)\left(\frac{\partial g(x_k)}{\partial x_k}\bigg|_{\bar{x}_k}\right) \mathrm{Covar}\,[x_i,x_k] \tag{3.3b}$$

where

$g(x_i)$ = random variable function of x_i variables

$\bar{x}_i$ = average of x_i

Higher orders of expectation of the random variable can also be computerd, for a complete description of the probability density distribution. In the following, it was assumed that the random variable is normally distributed and therefore, the higher order of expectations are not needed. Pavement performance, expressed in terms of number of coverages, is not likely normally distributed but rather closer to a log-normal distribution. The number of coverages can not be chosen as the independent variable. The next variable in the design sequence is the design factor whose distribution can be approximated by the normal one. Through a change of variable, this will lead to log-normal distribution for the number of coverages. Therefore, $g(x_i)$ in equation 3.3 represents DF, and x_i are the design variables namely MR, h_1, E_1, h_2, E_2, E_3 and LOAD. The design factor is expressed, for each aircraft:

$$DF = \frac{MR}{\sigma} \tag{3.4a}$$

$$\sigma = \sigma\,(h_1,\ E_1/E_{comp}) \tag{3.4b}$$

$$\left(\frac{E_{comp}}{E_3}\right)_n = f_1\left(h_1, h_2, \frac{E_1}{E_3}, \frac{E_2}{E_3}\right)^{f_2} \tag{3.4c}$$

where the functions σ, f_1 and f_2 are given in equations 3.2, 2.7 and 2.9 respectively.

Assuming that only E_1 and MR are correlated and substituting equations

3.4 into 3.3b lead to the following expression of the coefficient of variation of DF:

$$CV^2[DF] = CV^2[MR] + \sum_{i=1}^{n} \left(\frac{x_i}{\sigma} \frac{\partial\sigma}{\partial x_i} \bigg|_{\tilde{x}_i} . CV\ [x_i] \right)^2 +$$

$$-2\ \frac{E_1}{\sigma}\ \frac{\partial\sigma}{\partial E_1}\ \rho[E_1, MR] . CV[MR]\ .\ CV[E_1] \qquad (3.5)$$

where $x_i = h_1, h_2, E_1, E_2, E_3$

Computation of the derivatives of DF with respect to x_i for a given aircraft are as follows:

$$\frac{\partial DF}{\partial h_1} = -\frac{MR}{\sigma^2}\frac{\partial\sigma}{\partial h_1} \qquad (3.6a)$$

$$\frac{\partial DF}{\partial h_2} = -\frac{MR}{\sigma^2} \cdot \frac{\partial\sigma}{\partial h_2} \qquad (3.6b)$$

$$\frac{\partial DF}{\partial E_1} = -\frac{MR}{\sigma^2} \cdot \frac{\partial\sigma}{\partial E_1} \qquad (3.6c)$$

$$\frac{\partial DF}{\partial E_2} = -\frac{MR}{\sigma^2}\frac{\partial\sigma}{\partial E_{comp}} \cdot \frac{\partial E_{comp}}{\partial E_2} \qquad (3.6d)$$

$$\frac{\partial DF}{\partial E_3} = -\frac{MR}{\sigma^2} \cdot \frac{\partial\sigma}{\partial E_{comp}} \cdot \frac{\partial E_{comp}}{\partial E_3} \qquad (3.6e)$$

Using equations 3.2 and 2.8, the following final equations are obtained:

$$\frac{\partial\sigma}{\partial h_1} = \frac{\partial\sigma}{\partial h_1}\bigg|_{E_{comp}} - \frac{\partial\sigma}{\partial(E_1/E_{comp})}\left[\frac{E_1E_3}{E_{comp}^2}\cdot\frac{\partial(E_{comp}/E_3)}{\partial h_1}\right] \quad (3.7a)$$

$$\frac{\partial\sigma}{\partial h_2} = -\frac{E_1E_3}{E_{comp}}\cdot\frac{\partial\sigma}{\partial(E_1/E_{comp})}\cdot\frac{\partial(E_{comp}/E_3)}{\partial h_2} \quad (3.7b)$$

$$\frac{\partial\sigma}{\partial E_1} = \frac{1}{E_{comp}}\frac{\partial\sigma}{\partial(E_1/E_{comp})}\left[1 - \frac{E_1}{E_{comp}}\frac{\partial(E_{comp}/E_3)}{\partial(E_1/E_3)}\right] \quad (3.7c)$$

$$\frac{\partial\sigma}{\partial E_2} = \frac{-E_1}{E_{comp}^2}\frac{\partial\sigma}{\partial(E_1/E_{comp})}\cdot\frac{\partial(E_{comp}/E_3)}{\partial(E_2/E_3)} \quad (3.7d)$$

$$\frac{\partial\sigma}{\partial E_3} = -\frac{E_1}{E_{comp}^2}\cdot\frac{\partial\sigma}{\partial(E_1/E_{comp})}\left\{1 - \frac{1}{E_3}\left[E_1\frac{\partial(E_{comp}/E_3)}{\partial(E_1/E_3)} + E_2\frac{\partial(E_{comp}/E_2)}{\partial(E_2/E_3)}\right]\right\} \quad (3.7e)$$

and

$$\frac{\partial\sigma}{\partial h_1}\bigg|_{E_{comp}} = a_H + a_{EH}(E_1/E_{comp}) + a_{E2H}(E_1/E_{comp})^2 +$$

$$-\frac{\ell n\beta}{h_1^{\ 2}}\left[a_1 + \frac{2a_3}{\beta} - \frac{2a_4}{h_1} + 2a_4\frac{\ell n\beta}{h_1}\right] + \frac{1}{h_1}\left[\frac{a_1}{h_1} - \frac{a_2}{\beta} + \frac{a_3}{h_1\beta} - \frac{2a_5}{\beta^2}\right] \quad (3.8a)$$

$$\frac{\partial\sigma}{\partial(E_1/E_{comp})} = a_E + a_{EH}\,h_1 + 2a_{E2H}\,h_1\,E_1/E_{comp} +$$

$$\frac{1}{3h_1(E_1/E_{comp})}\left[a_1 - \frac{a_2h_1}{\beta} + \frac{a_3}{\beta}(1-\ell n\beta) + \frac{2a_4\ell n\beta}{h_1} - \frac{2a_5h_1}{\beta^2}\right] \quad (3.8b)$$

$$\frac{\partial(E_{comp}/E_3)}{\partial h_1} = a_n\left(\frac{E_{comp}}{E_3}\right)_n \cdot \frac{\ell n\alpha}{h_1}\,[a_1'\ell n\alpha + 2a_4'\,\ell nh_1] \quad (3.8c)$$

$$\frac{\partial(E_{comp}/E_3)}{\partial h_2} = a_n\left(\frac{E_{comp}}{E_3}\right)_n \cdot \left\{(\ell n\alpha)^2\left(\frac{a_2'}{h_2+1}\right) + \right.$$

$$2\ell n\alpha\left[a_1'\ell nh_1 + a_2'\ell n(h_2+1) + a_3'\ell n\frac{E_2}{E_3}\right] +$$

$$\left.\frac{1}{h_2+1}\left[a_4'(\ell nh_1)^2 + a_5'\left(\ell n\frac{E_1}{E_3}\right)^2\right]\right\} \quad (3.8d)$$

$$\frac{\partial(E_{comp}/E_3)}{\partial(E_1/E_3)} = a_n\left(\frac{E_{comp}}{E_3}\right)_n\left[2a_5'\ell n\alpha\ell n\frac{E_1}{E_3}\,/\,(E_1/E_3)\right] \quad (3.8e)$$

$$\frac{\partial(E_{comp}/E_3)}{\partial(E_2/E_3)} = a_n\left(\frac{E_{comp}}{E_3}\right)_n\left\{(\ell n\alpha)^2\left(\frac{a_3'}{E_2/E_3}\right) + \right.$$

$$\frac{2}{3}\,\ell n\alpha\left[a_1'\ell nh_1 + a_2'\ell n(h_2+1) + a_3\ell n\frac{E_2}{E_3}\right] +$$

$$\left.\frac{1}{3E_2/E_3}\left[a_4'\,(\ell nh_1)^2 + a_5'\left(\ell n\,\frac{E_1}{E_2}\right)^2\right]\right\} \quad (3.8f)$$

The probabilistic analysis of PCC airfield designs, based upon multilayered elastic theory, relies upon the determination of the probability density distribution of N for a given aircraft type and given pavement geometry. The general solution sequence is summarized below:

(1) Computer average stress ($\bar{\sigma}$) using equation 3.2 and average values of the design variables. (BISAR could have been used in the calculation of $\bar{\sigma}$; however, equation 3.2 has been used within the program in order to reduce the additional computer time which would have been required to run BISAR. BISAR is used later in the program if a corrected composite modulus is required.)

(2) Compute average DF (DF) as:

$$\overline{DF} = \overline{MR}/\overline{\sigma} \tag{3.9}$$

(3) Compute the variance of DF using equation 3.5 and the derivations in equations 3.6 to 3.8.

(4) Using $\overline{DF}$ and Var[DF], compute the cumulative distribution of DF from the normal distribution. Subdivide the interval of DF $\pm$ 3σ into say 30 DF_y values and compute the cumulative distribution P_y ($DF \leq DF_y$) corresponding to each DF_y.

(5) Compute N_y corresponding to DF_y using equation 2.3. The N_y is related to P_y, giving the cumulative distribution of number of coverages.

This scheme is implemented in a computer program (see Appendices I and II), and makes use of the regression equations developed for this purpose. However, it was shown that the degree of accuracy achieved in the prediction of the number of coverages depends upon the degree of accuracy in the composite modulus and stress computations. Because most of the error is due to the E_{comp} evaluation, a correction option for reducing the error was included in the program. The correction scheme, which corrects the value of E_{comp} to give an exact tensile stress is as follows:

(1) Input average design variables h_1, h_2, E_1, E_2 and E_3 and the design load using the format of the BISAR program.

(2) Compute maximum tensile stress $-\sigma'$ for the original layered system, using BISAR program.

(3) Using Newton-Raphson method for determination of roots of equation, compute $\overline{E}_{comp}$ corresponding to

$$\sigma - \sigma' = 0 \tag{3.10a}$$

where σ is given by equation 3.2.

(4) Correct E_{comp} as computed from equation 2.7 by adding a constant term

$$a_o = \overline{E}_{comp} - E_{comp} \tag{3.10b}$$

The correction scheme corresponds to a translation of the E_{comp} -function. This procedure does not affect the derivative of E_{comp} with respect to design variables. It is simple and requires only one run of the BISAR program.

The application of the probabilistic analysis and correction schemes will be illustrated through the example runs.

The Simulation Approach

The probabilistic analysis with the simulation approach includes 300 computation runs where the independent variables are randomly generated and the dependent variables DF and N, computed using equation 2.7, 2.3, 2.1 and 3.2. The number of runs (300) was choosen in order to insure a good description of the probability density distribution of both independent and dependent variables. In the following computations, it was assumed that the independent variables are normally distributed.

It should be noted that this assumption is not mandatory, and any given density distribution can be easily simulated. The generation of the random variables was made in the following steps, for each independent variable:

(1) generate $n = 12$ random decimal numbers with a uniform distribution from 0.0 to 1.0, a mean of 0.5 and a standard deviation of $\sigma = 1/\sqrt{12}$.

(2) From the Central Limit Theorem, the random variable defined as:

$$k = \frac{\sigma}{\sqrt{n/12}} \sum_{i=1}^{n} r_i + \left(\mu - \frac{n}{2} \frac{\sigma}{\sqrt{n/12}}\right) \tag{3.11a}$$

is a random observation from an approximately normal distribution with mean μ and standard deviation σ. Choosing $n = 12$ for computation convenience, and $\mu = 0.0$ and $\sigma = 1.0$ for the standard normal distribution, leads to:

$$k = \left(\sum_{i=1}^{12} r_i\right) - 6 \tag{3.11b}$$

(3) In order to insure closeness to a normal distribution, any set of 300-k whose standard deviation, skewness and kurtosis coefficients is outside the range of 0.95 to 1.05, -0.15 to 0.15 and 2.5 to 3.5 respectively is disregarded, and a new number set is generated.

(4) generate the independent random variables x_j from:

$$x_j = \bar{x}\,(1. + k_j \cdot CV[x]) \tag{3.12}$$

where k_j = given by equation 3.11b

$CV[x]$ = coefficient of variation of x

The above generated variables are substituted into equation 2.7, 2.3, 2.1 and 3.2 to give 300 random variables of DF_j and N_j . They are then

analyzed to derive their mean, standard deviation, and frequency distribution. Results of these analyses will be presented in the next paragraph and compared with those of the approximate closed-form approach.

It should be noted that simulation approach uses the same option for correction E_{comp} as in the approximate closed-form one. In addition, whenever h_2 (the base/subbase layer thickness) equals zero and E_2/E_3 (the ratio of base and subgrade moduli) equals one, the E_{comp} takes the values of the subgrade modulus.

Run Examples

Composite Modulus Correction

Figures 3.1a to 3.1e show the input data and output results of a run example, illustrating the composite modulus correction methodology. The case dealt with is: AG-4 loading and a combination of pavement geometry variables corresponding to 20 percent deviation in the composite modulus evaluation. The exact computed stress from the BISAR program using the original pavement is 479.62 psi. The composite modulus computed from equation 2.7 for the given pavement geometry is 34777 psi, while a value of 28980 psi is necessary in order to get the correct stress of 479.62 from equation 3.2. Therefore a correction factor of $a_o = -5796$ psi is used to reduce the composite modulus evaluated from equation 2.7. When this correction factor is applied in the approximate closed-form approach, it leads to exact values of the average stress, design factor and number of coverages.

The corrected modulus replaces that evaluated from equation 2.7 in all subsequent computations. It should be noted that the correction has little effect on the derivatives evaluated from the regression equations.

```
UZAR*LL(1).DALORK(3)
          4       RUN  EXAMPLE  WITH AG-4
          7
             7.0      0.02  4000000.       0.10
             4.00     0.10  600000.0       0.20
         12000.0      0.20
           700.       0.10       0.00
          0 1
           RUN EXAMPLE  COMPOSITE MODULUS CORRECTION
          1
          4  11
            4.0E+06     0.2E+00     0.7E+01     1.0E+03
            8.0E+05     0.2E+00     0.6E+01
            1.2E+08     0.9E+00     0.8E+02
            1.0E+04     0.8E+00
          2
        STRS  0.871875E+02 11.2778E+00     4.0E+01     0.0E+00
        STRS  0.871875E+02 11.2778E+00     0.0E+00     0.0E+00
                 SRRSTTSZZSFTSRZSTZ
          1
          1      0.0E+01     0.0E+00     0.7E+01
          4       RUN  EXAMPLE  WITH AG-4
          7
             7.0      0.02  4000000.       0.10
             6.00     0.10  600000.0       0.20
         12000.0      0.20
           700.       0.10       0.00
          1 1
           RUN EXAMPLE  COMPOSITE MODULUS CORRECTION
          1
          4  11
            4.0E+06     0.2E+00     0.7E+01     1.0E+03
            8.0E+05     0.2E+00     0.6E+01
            1.2E+08     0.8E+00     2.8E+02
            1.0E+04     0.8E+00
          2
        STRS  0.871875E+02 11.2778E+00     4.0E+01     0.0E+00
        STRS  0.871875E+02 11.2778E+00     0.0E+00     0.0E+00
                 SFFSTTSZZSFTSFZSTZ
          1
          1      0.0E+00     0.0E+00     0.7E+01
**

@COPY,A LL.
 1 APS

@XQT,FO LL.AFS
  AG- 4        RUN  EXAMPLE  WITH AG-4
```

Figure 3.1a INPUT DATA FOR RUN EXAMPLE AG-4

SYSTEM NUMBER 1

LAYER NUMBER	CALCULATION METHOD	YOUNG'S MODULUS	POISSON'S RATIO	THICKNESS	REDUCED SPRINGCOEFFL
1	SMOOTH	.8[illegible]+07	.2[illegible]+00	.7000+01	.1000+[illegible]
2	SMOOTH	.6[illegible]+06	.2[illegible]+00	.6000+01	.0000
3	SMOOTH	.12[illegible]+05	.4[illegible]+00	.2400+0[illegible]	.0000
4		.1[illegible]+07	.4[illegible]+00		

LOAD NUMBER	NORMAL STRESS	SHEAR STRESS	RADIUS OF LOADED AREA	LOAD - POSITION X	LOAD - POSITION Y	SHEAR DIRECTION
1	.8719+02	.0000	.1123+02	.4000+02	.0000	.0000
2	.9719+02	.0100	.1123+02	.[illegible]100	.[illegible]	.0000

Figure 3.1b SYSTEM DESCRIPTION - OUTPUT FOR RUN EXAMPLE

```
                                          POSITION NUMBER  1
                                           LAYER NUMBER   1
                                            COORDINATES
                                     X            Y            Z
                                   .0000        .0000       .7000+01
                    DISTANCE TO LOAD-AXIS( 1)                                    THETA
                          .8000+02                                                .3142+01
 STRESSES
  RADIAL          TANGENTIAL          VERTICAL          RAD./TANG.          RAD./VERT.          TANG./VERT.
 -.8710+02           .2992+02         -.6931+03            .0000            -.2103+01              .0000
 STRAINS
  RADIAL          TANGENTIAL          VERTICAL          RAD./TANG.          RAD./VERT.          TANG./VERT.
 -.1720-08           .9669-08          .8858-06                             -.6310-06
                    DISTANCE TO LOAD-AXIS( 2)                                    THETA
                          .0000                                                   .0000
 STRESSES
  RADIAL          TANGENTIAL          VERTICAL          RAD./TANG.          RAD./VERT.          TANG./VERT.
  .8897+03           .8897+03         -.1951+02            .0000              .0000                .0000
 STRAINS
  RADIAL          TANGENTIAL          VERTICAL          RAD./TANG.          RAD./VERT.          TANG./VERT.
  .9692-08           .9692-08         -.8985-08                               .0000
                         XX          YY          ZZ          YZ          XZ          XY          UX          UY          UZ
T O T A L  S T R E S S  .887+03     .880+03    -.202+02     .000        .210+01     .000
T O T A L  S T R A I N  .787-08     .101-01    -.899-08     .000        .631-06     .000
T O T A L  D I S P L A C E M E N T
P R I N C I P A L  V A L U E S  A N D  D I R E C T I O N S  O F  T O T A L  S T R E S S E S  A N D  S T R A I N S
             NORMAL        NORMAL         SHEAR         SHEAR             X              Y              Z
             STRESS        STRAIN         STRESS        STRAIN        COMPONENT      COMPONENT      COMPONENT
MAXIMUP      .880+03       .101-01                                       .000          1.000           .000
MINIMAX      .897+03       .787-08                                      1.000           .000           .005
MINIMUP     -.202+02      -.898-08                                      -.005           .000          1.000
MAXIMUM                                   .253+03       .750-08          .001           .707          -.707
             .290+03                                                    -.003           .707           .707
MINIMAX                                   .213+03       .690-08          .711           .000          -.708
             .199+03                                                     .708           .000           .711
MINIMUM                                   .165+02       .110-08         -.707           .703          -.003
             .887+03                                                     .707           .707           .003
             STRAIN ENERGY      .9962-01
STRAIN ENERGY OF DISTORTION     .2187-01
```

Figure 3.1c   STRESS RESULTS - OUTPUT FOR RUN EXAMPLE

AD-A138 212 DEVELOPMENT OF PROBABILISTIC RIGID PAVEMENT DESIGN METHODOLOGIES FOR MILI..(U) MARYLAND UNIV COLLEGE PARK DEPT OF CIVIL ENGINEERING M W WITCZAK ET AL. DEC 83 3/3

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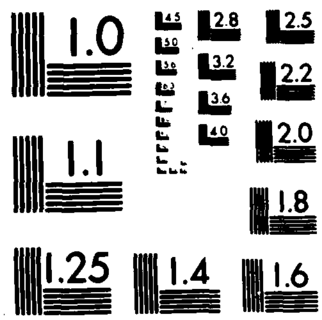

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

```
PREVIOUS ECONF=    34777. CORRECTED ECONF=    28981. CORRECTION =    -5796.
 CORRECTED AVERAGE STRESS FROM BISAR =         479.419

CONCRETE  MODULUS  =        4000000. PSI     CV=            .100
BASE      MODULUS  =        [illegible]. PSI     CV=            .200
SUBGRADE  MODULUS  =          12000. PSI     CV=            .200
SLAB   THICKNESS   =           7.00  IN.     CV=            .020
BASE   THICKNESS   =           6.00  IN      CV=            .100
MODULUS OF RUPTURE =           700. PSI      CV=            .100
REGRESSION COEFFICIENT=                               .000

AVERAGE  STRESS                    =          479.46 PSI
AVERAGE  DESIGN FACTOR             =            1.46
COEFFICIENT OF VARIATION OF DF=               .1229
AVERAGE NUMBER OF COVERAGES        =            284.
SORT WEIGHTING FACTOR FOR H1       =         -.89252
SORT WEIGHTING FACTOR FOR H2       =         -.36855
SORT WEIGHTING FACTOR FOR E1       =          .33878
SORT WEIGHTING FACTOR FOR E2       =         -.15273
SORT WEIGHTING FACTOR FOR ES       =         -.18575
RELIABILITY    DESIGN FACTOR    N. OF COVERAGES
     .9987          .9213              9.
     .9974          .9572             11.
     .9953          .9931             14.
     .9918         1.0290             17.
     .9861         1.0648             22.
     .9772         1.1007             28.
     .9641         1.1365             35.
     .9452         1.1724             44.
     .9192         1.2083             56.
     .8849         1.2441             70.
     .8413         1.2800             89.
     .7881         1.3158            112.
     .7257         1.3518            141.
     .6554         1.3876            178.
     .5793         1.4235            225.
     .5000         1.4594            284.
     .4207         1.4952            358.
     .3446         1.5311            452.
     .2743         1.5670            570.
     .2119         1.6028            719.
     .1587         1.6387            908.
     .1151         1.6746           1146.
     .0808         1.7104           1446.
     .0548         1.7463           1825.
     .0359         1.7822           2303.
     .0228         1.8180           2907.
     .0139         1.8539           3669.
     .0082         1.8898           4630.
     .0047         1.9256           5843.
     .0026         1.9615           7378.
     .0013         1.9974           9307.
 AE- 8        RLA  EXAMPLE  WITH AE-8
```

Figure 3.1d OUTPUT RESULTS FROM APPROXIMATE CLOSED FORM PROBABILISTIC SOLUTION

```
PREVIOUS ECOMP=    34777. CORRECTED ECOMP=    28980. CORRECTION =    -5796.
 CORRECTED AVERAGE STRESS FROM EISAR =       479.619

CONCRETE  MODULUS  =       4000000. PSI    CV=          .100
BASE      MODULUS  =        800000. PSI    CV=          .200
SUBGRADE  MODULUS  =         12000. PSI    CV=          .200
SLAB   THICKNESS   =          7.00  IN.    CV=          .020
BASE   THICKNESS   =          6.00  IN     CV=          .100
MODULUS OF RUPTURE =          700. PSI     CV=          .100
REGRESSION COEFFICIENT=                              .000


STRESS FREQUENCY  DISTRIBUTION

MINIMUM                          =        374.2829
MAXIMUM                          =        637.8759
AVERAGE                          =        483.7479
VARIANCE                         =       1568.9348
COEFFICIENT  OF  VARIATION       =          .0818

     X         Y      X         Y      X         Y      X         Y      X         Y
  .3787+03     2   .3875+03     0   .3962+03     0   .4050+03     1   .4139+03     6
  .4226+03     8   .4314+03    12   .4402+03    21   .4490+03    13   .4578+03    38
  .4665+03    21   .4753+03    27   .4841+03    20   .4929+03    22   .5017+03    25
  .5105+03    19   .5193+03    15   .5280+03     9   .5368+03    11   .5456+03     7
  .5544+03     7   .5632+03     2   .5720+03     5   .5808+03     2   .5896+03     0
  .5983+03     1   .6071+03     0   .6159+03     0   .6247+03     0   .6335+03     1
 DF  FREQUENCY  DISTRIBUTION

MINIMUM                          =          1.0188
MAXIMUM                          =          2.0929
AVERAGE                          =          1.4542
VARIANCE                         =           .0337
COEFFICIENT  OF  VARIATION       =           .1242

     X         Y      X         Y      X         Y      X         Y      X         Y
  .1032+01     2   .1068+01     8   .1104+01     2   .1140+01     6   .1176+01     7
  .1212+01     9   .1248+01    11   .1284+01    20   .1320+01    23   .1356+01    15
  .1392+01    27   .1428+01    21   .1464+01    21   .1500+01    25   .1536+01    21
  .1572+01    15   .1608+01    18   .1644+01    19   .1679+01    13   .1715+01    14
  .1751+01     5   .1787+01     1   .1823+01     4   .1859+01     0   .1895+01     1
  .1931+01     0   .1967+01     1   .2003+01     1   .2039+01     0   .2075+01     1

RELIABILITY  DESIGN FACTOR  N.OF COVERAGES
     .9933      1.0503             20.
     .9767      1.0863             25.
     .9700      1.1272             32.
     .9533      1.1587             40.
     .9267      1.1981             51.
     .8967      1.2301             64.
     .8600      1.2660             81.
     .7933      1.3029            102.
     .7167      1.3379            120.
     .6467      1.3718            147.
     .5900      1.4098            206.
     .5200      1.4458            260.
     .4500      1.4817            328.
     .3667      1.5177            418.
     .2967      1.5536            523.
     .2467      1.5896            660.
     .2000      1.6255            838.
     .1367      1.6615           1053.
     .0933      1.6974           1320.
     .0467      1.7334           1679.
     .0300      1.7693           2119.
     .0267      1.8053           2676.
     .0133      1.8412           3379.

     .0133      1.8772           4267.
     .0100      1.9131           5388.
     .0100      1.9491           6803.
     .0067      1.9850           8591.
     .0033      2.0210          10847.
     .0033      2.0569          13697.
     .0000      2.0929          17296.
```

Figure 3.1e   OUTPUT RESULTS FROM SIMULATION SOLUTION

When the correction factor is used in conjunction with the simulation approach - i.e., all 300 composite moduli evaluated are shifted by 5796 psi, regardless of their generated value, the average stress generated is 483.74 psi, higher than the one computed from BISAR program by 0.9 percent. It seems that the composite modulus correction scheme is adequate for reducing to minimum the errors associated with the use of regression equations.

Comparison of the Approximate Closed-Form and the Simulation Approaches

The errors involved in the approximate closed-form approach are due to the linear assumption used in the Taylor series expansion. Whenever the dependent variable is related to the independent variables through a non-linear function, some error will result in the evaluation of the average and of the variance of the variable. Therefore, the results should be checked and compared with other results obtained from a different approach which does not assume linearity. In this paragraph, the results of the approximate closed-form approach are compared to those of the simulation scheme which assumes that the independent variables are normally distributed, as achieved by the generation scheme presented above.

From Figures 3.1d and 3.1e, it is seen that the average and the coefficient of variation values of the design factor are very close: 1.46 and 0.1229 in the approximate closed-form solution, and 1.456 and 0.1262 in the simulation process. These values were obtained for realistic values of standard deviations of the independent variables. Figures 3.2a and 3.2b show the input data and the relationship between reliability and number of coverages of AG-13. It is seen that a slight deviation between the results exist at the distribution tails. The deviation which can be attributed to the number generation in the simulation process is considered negligible

```
13      RUN  EXAMPLE  WITH AG-13
 1
     20.0       0.02  8000000.       0.10
    10.00       0.10   800000.0      0.20
  12000.0       0.20
     700.       0.10      0.00
 1 1
 RUN EXAMPLE  COMPOSITE POPULUS CORRECTION
 1
 8  11
   8.0E+06      0.2E+00      2.0E+01      1.0E+03
   5.0E+08      0.2E+00      1.2E+01
   1.2E+08      0.8E+00      2.8E+08
   1.0E+06      0.8E+00
 8
STHS  2.3370735E+02  5.2190E+00    -3.7E+01      0.0E+00
STHS  2.3377779E+02  9.2149E+00     0.3E+00      2.0E+00
STHS  2.3370735E+02  5.2190E+00     6.2E+01      0.0E+00
STHS  2.3377779E+02  9.2199E+00     9.9E+01      0.0E+00
        SRRS11S72SR1SR2S12
 1
 1      0.0E+00     0.0E+00     2.0E+01
13      RUN  EXAMPLE  WITH AG-13
 1
     20.0       0.02  8000000.       0.10
    10.00       0.10   800000.0      0.20
  12000.0       0.20
     700.       0.10      0.00
 1 1
 RUN EXAMPLE  COMPOSITE POPULUS CORRECTION
 1
 8  11
   8.0E+06      0.2E+00      2.0E+01      1.0E+03
   5.0E+08      0.2E+00      1.2E+01
   1.2E+08      0.8E+00      2.8E+08
   1.0E+06      0.8E+00
 8
STHS  2.3370735E+02  5.2190E+00    -3.7E+01      0.0E+00
STHS  2.3377779E+02  9.2199E+00     0.3E+00      0.3E+00
STHS  2.3370735E+02  5.2190E+00     6.2E+01      0.0E+00
STHS  2.3377779E+02  9.2199E+00     9.7E+01      0.7E+00
        SRRS11S72SR1SR2S12
 1
 1      0.0E+00     0.0E+00     2.0E+01
```

Figure 3.2a INPUT DATA FOR RUN EXAMPLE AG-13

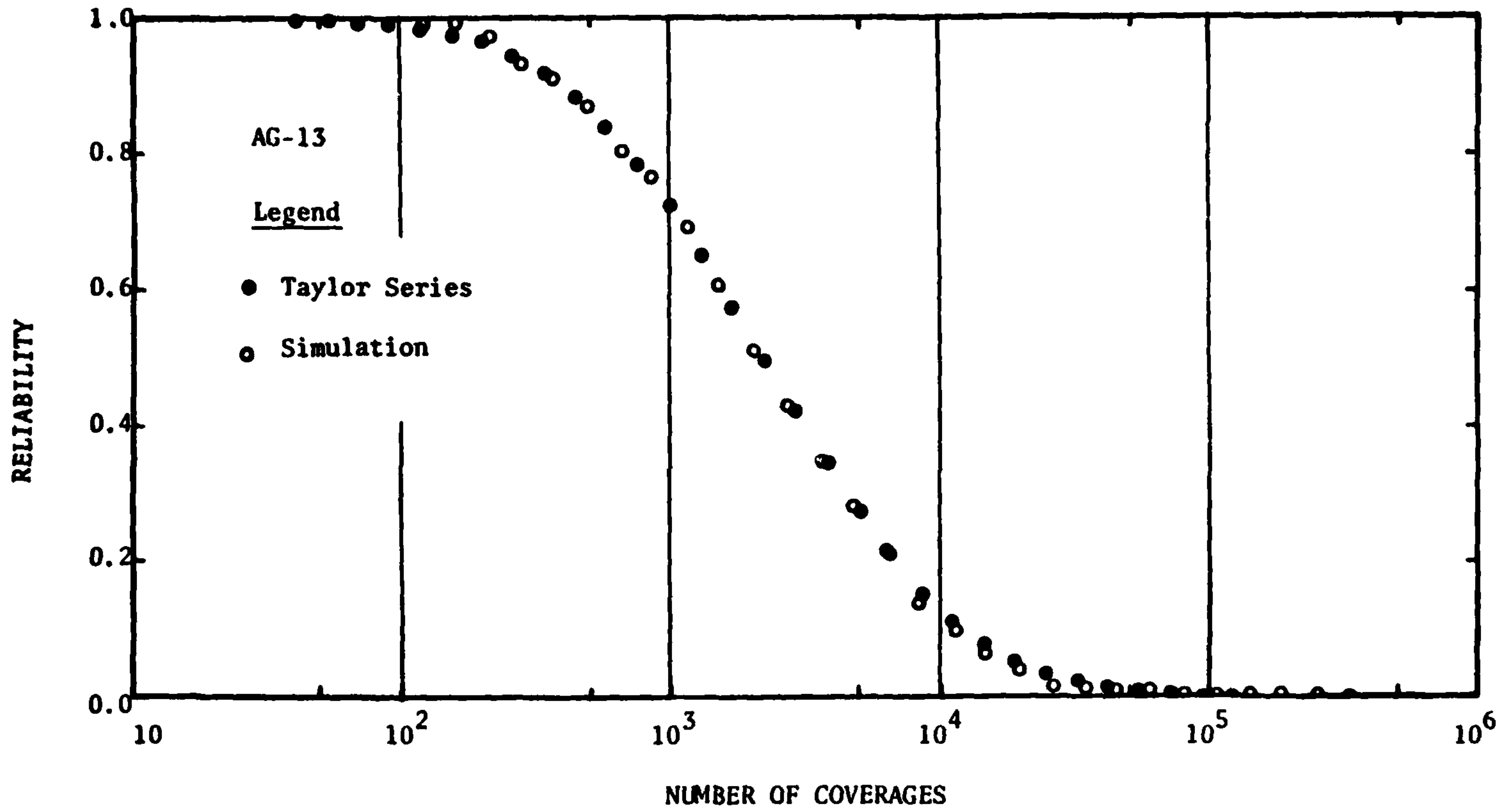

Figure 3.2b COMPARISON OF APPROXIMATE CLOSED FORM AND SIMULATION SOLUTIONS

from an engineering design point of view.

Table 3.3 summarizes study results of analysis for AG-2 using Taylor series expansion. The results are given in terms of weighting factors of the design variables in the coefficient of variation computation, i.e., when

$$CV^2[DF] = \sum_{i=1}^{n} (w_i \ CV[x_i])^2 \tag{3.13}$$

w_i is the weighting factor of x_i. Note that w_i equals one for x_i = MR. It is seen that the weighting factors of the concrete layer parameters (h_1 and E_1) increases as the other variables tend to increase the relative stiffness of the concrete layer. For example, w_{h_1} increases as h_1 increases, and as E_2, h_2 or E_3 decrease. As for the second and subgrade layers, their weighting factors increase as the other variables tend to increase the stress in the layer. For example, w_{E_3} increases as E_3 increases, and as h_1, h_2 or E_2 decrease. It appears that the weighting factor expresses the functional importance of the variable in the design. It should be noted that the results were derived without the correction option of the composite modulus and that the weighting factors are slightly different when the subgrade layer is subdivided in two layers of equal moduli. This is attributed to the form of the composite modulus regression function used. It is however stressed that the deviation from the homogeneous case is negligible.

In order to compare the results of Table 3.3 to those of the simulation process, only one variable at a time will be varied. Table 3.4a shows the results of simulating variation of h_1 only. It is seen that: (1) the coefficient of variation of the design factor DF is proportional to that of the concrete layer thickness, i.e., for the particular case studied, the linearity assumption seems correct; (2) the weighting factors

Table 3.3 Summary of Results for AG-2 using Taylor Series Expansion

E psi	h_1 in	h_2 in	E_2 psi	w_i -weighting factors of h_1	h_2	E_1	E_2	E_3
5000	8	0	-	-1.6444	-	0.1309	-	-0.1309
		6	20,000	-1.6212	-0.0420	0.1425	-0.0189	-0.1236
			50,000	-1.5872	-0.0769	0.1506	-0.0333	-0.1172
			200,000	-1.5153	-0.1525	0.1658	-0.0644	-0.1014
		12	20,000	-1.5519	-0.1256	0.1559	-0.0497	-0.1061
			50,000	-1.4915	-0.1907	0.1690	-0.0746	-0.0944
			200,000	-1.3620	-0.3302	0.1941	-0.1276	-0.0665
	10	0	-	-1.7200	-	0.1179	-	-0.1179
		6	20,000	-1.7112	-0.0280	0.1280	-0.0132	-0.1148
			50,000	-1.6847	-0.0566	0.1356	-0.0251	-0.1105
			200,000	-1.6279	-0.1200	0.1498	-0.0513	-0.0986
		12	20,000	-1.6574	-0.0960	0.1398	-0.0386	-0.1012
			50,000	-1.6092	-0.1511	0.1523	-0.0597	-0.0926
			200,000	-1.5078	-0.2682	0.1744	-0.1044	-0.0700
	12	0	-	-1.7980	-	0.1097	-	-0.1097
		6	20,000	-1.7989	-0.0192	0.1193	-0.0096	-0.1097
			50,000	-1.7789	-0.0437	0.1263	-0.0199	-0.1065
			200,000	-1.7348	-0.0990	0.1397	-0.0428	-0.0969
		12	20,000	-1.7571	-0.0769	0.1295	-0.0314	-0.0980
			50,000	-1.7198	-0.1254	0.1417	-0.0501	-0.0917
			200,000	-1.6404	-0.2290	0.1625	-0.0897	-0.0728
10,000	8	0	-	-1.6043	-	0.1519	-	-0.1519
		6	20,000	-1.5968	-0.0282	0.1606	-0.0134	-0.1472
			50,000	-1.5662	-0.0600	0.1668	-0.0265	-0.1402
			200,000	-1.5020	-0.1289	0.1784	-0.0549	-0.1235
		12	20,000	-1.5372	-0.1035	0.1694	-0.0415	-0.1279
			50,000	-1.4833	-0.1621	0.1797	-0.0639	-0.1159
			200,000	-1.3653	-0.2884	0.2017	-0.1119	-0.0897
	10	0	-	-1.6868	-	0.1406	-	-0.1406
		6	20,000	-1.6914	-0.0166	0.1484	-0.0087	-0.1398
			50,000	-1.6677	-0.0428	0.1536	-0.0196	-0.1342
			200,000	-1.6173	-0.1006	0.1636	-0.0435	-0.1201
		12	20,000	-1.6449	-0.0784	0.1553	-0.0321	-0.1233
			50,000	-1.6026	-0.1277	0.1641	-0.0510	-0.1133
			200,000	-1.5121	-0.2320	0.1815	-0.0908	-0.0907
	12	0	-	-1.7765	-	0.1337	-	-0.1337
		6	20,000	-1.7886	-0.0091	0.1414	-0.0057	-0.1358
			50,000	-1.7712	-0.0317	0.1462	-0.0151	-0.1311
			200,000	-1.7315	-0.0824	0.1547	-0.0362	-0.1186
		12	20,000	-1.7520	-0.0621	0.1469	-0.0260	-0.1209
			50,000	-1.7109	-0.1055	0.1549	-0.0427	-0.1122
			200,000	-1.6482	-0.1900	0.1698	-0.0777	-0.0922
20,000	8	0	-	-1.5651	-	0.1695	-	-0.1695
		6	20,000	-1.5717	-0.0140	0.1764	-0.0076	-0.1688
			50,000	-1.5464	-0.0416	0.1807	-0.0191	-0.1616
			200,000	-1.4867	-0.1041	0.1911	-0.0449	-0.1462
		12	20,000	-1.5304	-0.0782	0.1785	-0.0319	-0.1466
			50,000	-1.4731	-0.1327	0.1907	-0.0528	-0.1379
			200,000	-1.3625	-0.2487	0.2115	-0.0971	-0.1144
	10	0	-	-1.6655	-	0.1576	-	-0.1576
		6	20,000	-1.6727	-0.0047	0.1640	-0.0038	-0.1602
			50,000	-1.6530	-0.0271	0.1675	-0.0132	-0.1543
			200,000	-1.6068	-0.0789	0.1755	-0.0347	-0.1408
		12	20,000	-1.6377	-0.0573	0.1661	-0.0241	-0.1419
			50,000	-1.5980	-0.1021	0.1748	-0.0414	-0.1334
			200,000	-1.5124	-0.1867	0.1898	-0.0775	-0.1122
	12	0	-	-1.7560	-	0.1505	-	-0.1505
		6	20,000	-1.7800	-0.0012	0.1565	-0.0014	-0.1551
			50,000	-1.7651	-0.0177	0.1598	-0.0094	-0.1504
			200,000	-1.7286	-0.0627	0.1664	-0.0282	-0.1381
		12	20,000	-1.7503	-0.0435	0.1585	-0.0190	-0.1395
			50,000	-1.7188	-0.0824	0.1654	-0.0340	-0.1314
			200,000	-1.6535	-0.1645	0.1773	-0.0655	-0.1118

Table 3.4a Results of Simulation for AG-2 and Concrete Layer Thickness Weighting Factor

E_3 psi	h_1 in.	h_2 in.	E_2 psi	Values of CV[DF] for		Ratio	
				$CV[h_1]=$ 0.02 (1)	$CV[h_1]=$ 0.05 (2)	$\frac{(2)}{(1)}$	w_{h_1}
20,000	8	6	20,000	0.0321	0.0802	2.50	1.604
			50,000	0.0315	0.0787	"	1.574
			200,000	0.0302	0.0754	"	1.508
		12	20,000	0.0321	0.0802	"	1.604
			50,000	0.0299	0.0747	"	1.494
			200,000	0.0275	0.0687	"	1.374
	10	6	20,000	0.0339	0.0849	2.50	1.698
			50,000	0.0337	0.0843	"	1.686
			200,000	0.0327	0.0818	"	1.636
		12	20,000	0.0339	0.0849	"	1.698
			50,000	0.0325	0.0812	"	1.624
			200,000	0.0307	0.0768	"	1.536
	12	6	20,000	0.0360	0.0901	2.50	1.802
			50,000	0.0361	0.0902	"	1.804
			200,000	0.0353	0.0883	"	1.766
		12	20,000	0.0360	0.0901	"	1.802
			50,000	0.0351	0.0877	"	1.754
			200,000	0.0337	0.0843	"	1.686

are slightly higher, by about 3 percent in the simulation process than in the approximate closed-form solution. This is negligible for engineering practical purposes.

Table 3.4b shows the results of the simulation for the subgrade modulus weighting factor. It is seen that (1) increasing the coefficient of variation of the subgrade modulus from 0.10 to 0.25 increases the coefficient of variation of the design factor by 2.60, a slight deviation from linearity; (2) the weighting factors are higher in the simulation than in the Taylor series expansion results (up to 15 percent). While this discrepancy is quite high for the individual weighting factor, its effect on the coefficient of variation of the design factor with variability of all design parameters will be attenuated in the summation of the squared contribution (see equation 3.3).

It appears from the above that the results of reliability will be similar for the Taylor series expansion and simulation solution. The assumed linearity (or first order expansion) of the function seems to be supported by the simulation approach.

Example

The approximate approach was used to develop reliability-number of coverages curves for different thicknesses of the concrete layer (18, 20, 22 and 24 inch), see Figure 3.3a. Thickness - number of coverages curves were then developed for different reliability levels (0.9, 0.8, 0.7, 0.6 and 0.5) and shown in Figure 3.3b. This run example will serve to find out the reliability level used in the current design. The analysis is made for two values of percentile (0.85 and 0.90) for computing the design parameters in the current "deterministic" method. The results are summarized in Table 3.5 where:

Table 3.4b Results of Simulation for AG-2 and Subgrade Modulus Weighting Factor

E_3 psi	h_1 in	h_2 in	E_2 psi	Values of CV[DF] for CV[E_3]= (1)	CV[E_3]= (2)	Ratio $\frac{(2)}{(4)}$	w_{E_3}
10,000	8	6	20,000	0.0141	0.0367	2.60	0.1468
			50,000	0.0143	0.0370	2.59	0.1480
			200,000	0.0136	0.0353	2.60	0.1412
		12	20,000	0.0127	0.0331	2.61	0.1324
			50,000	0.0121	0.0315	2.60	0.1260
			200,000	0.0101	0.0262	2.59	0.1048
	10	6	20,000	0.0134	0.0347	2.59	0.1388
			50,000	0.0136	0.0352	2.59	0.1408
			200,000	0.0131	0.0340	2.60	0.1360
		12	20,000	0.0122	0.0318	2.61	0.1272
			50,000	0.0118	0.0305	2.58	0.1220
			200,000	0.0101	0.0261	2.58	0.1044
	12	6	20,000	0.0130	0.0336	2.58	0.1344
			50,000	0.0132	0.0342	2.59	0.1368
			200,000	0.0128	0.0333	2.60	0.1332
		12	20,000	0.0120	0.0310	2.58	0.1240
			50,000	0.0116	0.0301	2.59	0.1204
			200,000	0.0101	0.0261	2.58	0.1044

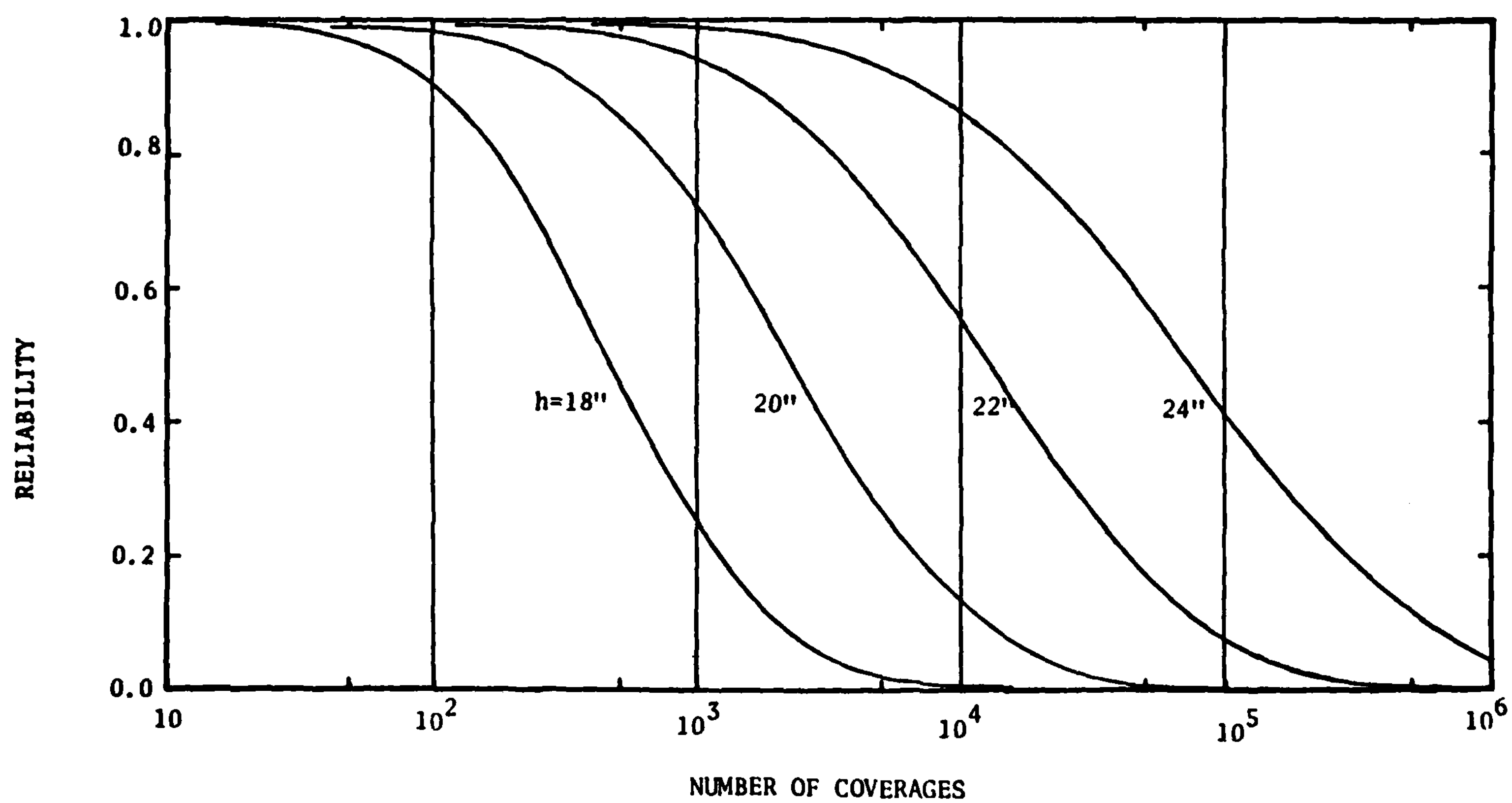

Figure 3.3a RELIABILITY - NUMBER OF COVERAGES RELATIONSHIPS FOR AG-13 AND DIFFERENT CONCRETE LAYER THICKNESSES

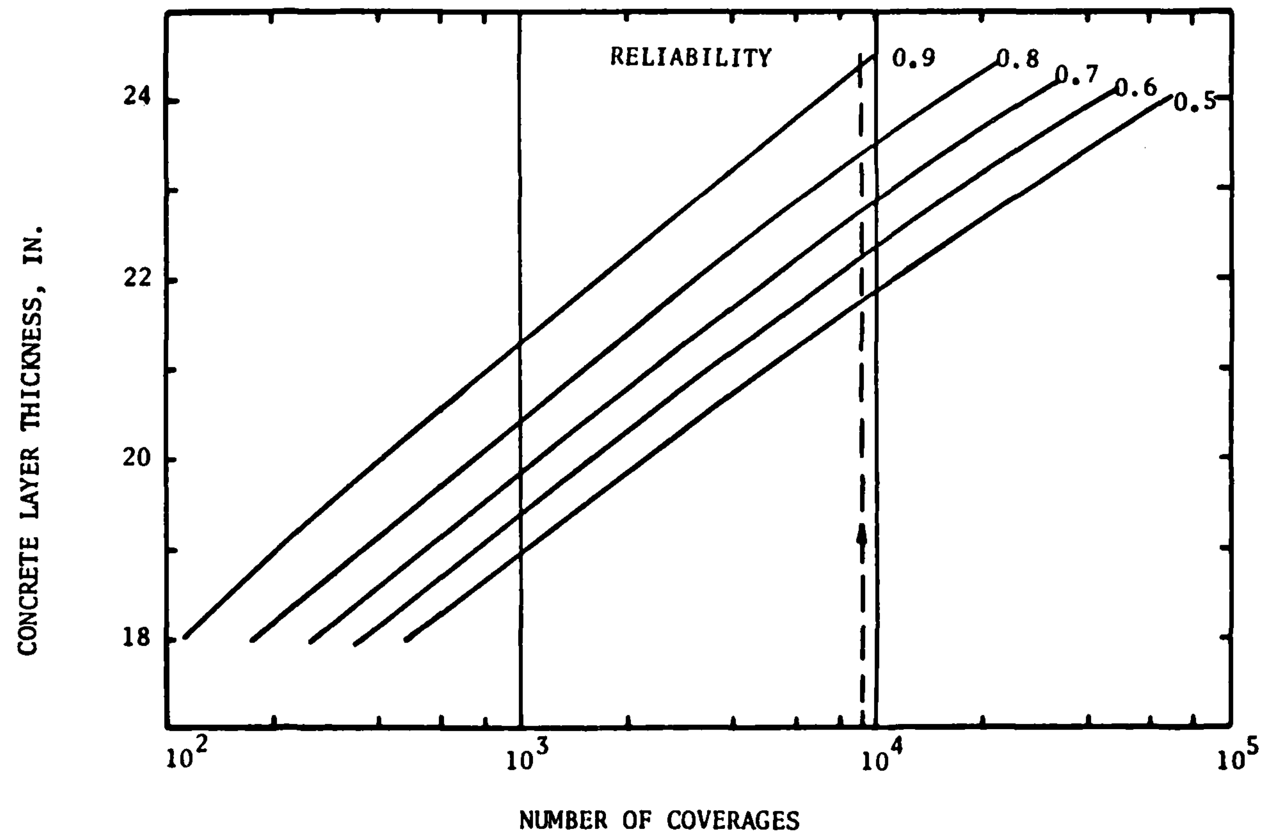

Figure 3.3b CONCRETE LAYER THICKNESS - NUMBER OF COVERAGES RELATIONSHIPS FOR AG-13 AND DIFFERENT RELIABILITY LEVELS

Table 3.5 Summary of Pavement Parameters in the Current Design Method

	Percentile Value	
	0.85	0.90
$K\alpha$ in normal distribution	1.03	1.30
Subgrade modulus of elasticity, psi	9,600.	8,900.
Base modulus of elasticity, psi	40,000.	37,000.
Subgrade modulus of reaction, pci	100.	94.
Composite subgrade modulus of reaction, pci	200.	180.
Modulus of rupture of concrete, psi	630.	610.
Required Thickness, in	24.2	25.1

$K_\alpha = \frac{a-\mu}{\sigma}$, where: a = value of random width
μ = the mean value
σ = the standard deviation

(a) The subgrade and the base modulus of elasticity, and the modulus of rupture are computed using:

$$x_i = \bar{x}_i \ (1 - K_\alpha \ CV[x_i]) \qquad (3.14)$$

(b) The average values of the variables and their coefficient of variation are given in Figure 3.2a.

(c) The subgrade modulus of reaction is computed from (Parker et al., (1)):

$$\log k = (\log M_R - 1.415)/1.284 \qquad (3.15)$$

(d) The composite subgrade modulus of reaction is derived from Figure 2.3.

(e) The required thickness is calculated using design curves in (1). The thickness corresponds to traffic area A and 9200 coverages.

Entering Figure 3.3b with N = 9200 coverages and h = 24.2 and 25.1 in., the corresponding reliability levels of 0.88 and 0.95 are derived. These values give an idea of the reliability levels used today in the design procedure.

Chapter 4

SUMMARY

The design procedure of military rigid airfield pavements developed by Parker et al (1) is expressed in this volume in probabilistic and reliability terms. The procedure, based on the multi-layer elastic theory necessitated further developments in order to make the analysis feasible. Two major investigations were conducted: (1) Evaluation of the Composite Modulus of Elasticity of the layered (subbase/subgrade) system underneath the rigid pavement and (2) Evaluation of the maximum tensile stress at the bottom of the concrete layer for each of 13 aircraft types: USAF Classification AG-1 to 13.

Valuable results were derived from the investigation of the composite modulus. It was found that:

(1) The composite modulus depends not only upon the design parameters of the layered (subbase/subgrade) system underneath the rigid pavement, but also upon the pavement thickness (for constant elastic properties of the concrete) and the loading conditions (number of wheels in the gear).

(2) The effect of deviations in the composite modulus evaluation on pavement performance (expressed in terms of predicted allowable number of coverages) is quite substantial. The error in the evaluation of number of coverages is about 1.2 to 5 times that achieved in the evaluation of the composite modulus. The lower range corresponds to light load aircraft and traffic whereas the upper range corresponds to heavy load aircraft and traffic. This result emphasizes the need for an accurate determination of the composite modulus.

(3) The accuracy achieved in a correlation between the composite modulus and the design parameters is excellent (R^2 = 0.994 and SE = 8%). However, in the case of heavy load aircraft and larger number of coverages, this (accuracy) is insufficient for both design purposes and probabilistic/reliability analyses. Therefore, a correction methodology is developed to reduce the error to minimum.

A regression equation of the maximum tensile stress at the bottom of the concrete layer is derived for each of 13 aircraft types. The equation is highly accurate, allowing its use without any correction for both design purposes and probabilistic/reliability analyses.

The above derivations of the composite modulus and of the maximum tensile stress are included in a computer program for probabilistic/reliability analysis of rigid pavements. Both the approximate closed form (Taylor series expansion) and the simulation solutions are implemented. The computer program can be used:

(1) In the analysis in probabilistic/reliability terms of a specific pavement structure and loading aircraft, for a given material properties and variabilities. The design parameters (means and coefficient of variations) serve as input to the computer program which produces values of number of coverages and their corresponding reliability levels.

(2) In the design of a rigid pavement at a specific reliability level, given all design parameters. Several solutions of the above under (1) for different pavement structures must be conducted, and the requested design is derived.

Several example runs of the computer program are presented, illustrating (a) the use of the correction procedure in the evaluation of the composite modulus. It is suggested to call the procedure in the

case of medium and heavy load and traffic, (b) the simultude of the results obtained with the approximate closed form and the simulation solutions and (c) the interpretation of the current deterministic design procedure in probabilistic/reliability terms.

LIST OF REFERENCES

1. Parker, F., Jr., Barker, W.R., Gunkel, R.C. and Odom, E.C., "Development of a Structural Design Procedure for Rigid Airport Pavements", U.S. Army Eng., W.E.S. Final Report, April 1979.

2. Army T.M., "Rigid Pavement for Airfields Other than Army", Army T.M. 5.824-3, and Air Force AFM 88-6, Chap. 3, Aug. 1979.

3. Ulery, H.H., Letter to M.W. Witczak, August 1982.

APPENDIX I

USER'S GUIDE

Card 1 (I2, 19A4)

1-2	IG	- aircraft group number
3-78	HED	- text for title

Card 2 (I2)

1-2	NLA	- number of layers (=3 when base-subbase layer exists)

Card 3 (4F10.0)

1-10	H1AV	- average thickness of concrete layer, inch.
11-20	H1STD	- coefficient of variation of concrete layer thickness
21-30	E1AV	- average modulus of elasticity of concrete, psi.
31-40	E1STD	- coefficient of variation of concrete modulus of elasticity

Card 4 (4F10.0) required if NLA = 3

1-10	H2AV	- average thickness of base-subbase layer, inch.
11-20	H2STD	- coefficient of variation of base-subbase layer thickness
21-30	E2AV	- modulus of elasticity of base-subbase material, psi.
31-40	E2STD	- coefficient of variation of base-subbase modulus of elasticity

Card 5 (2F10.0)

1-10	ES	- subgrade modulus of elasticity, psi.
11-20	ESSTD	- coefficient of variation of subgrade modulus of elasticity

Card 6 (3F10.0)

1-10	MR	- average concrete modulus of rupture, psi.
11-20	MRSTD	- coefficient of variation of concrete modulus of rupture
21-30	ROEMR	- regression coefficient between modulus of elasticity and modulus of rupture of concrete

<u>Card 7</u> (2I2)

1-2 IRAN - flag ≤ 0 for running Taylor series expansion approach

> 0 for running simulation approach

3-4 IOPT - flag $= 0$ no correction for composite modulus computations

$\neq 0$ run BISAR program for correcting composite modulus

<u>Card 8</u> required if IOPT $\neq 0$

Input data required by BISAR program

<u>Note</u> Several problems (Cards 1 - 7) can be run in sequence.

APPENDIX II

PROGRAM LISTING

Appendix II is a copyrighted program listing. Information on the program can be obtained from the authors of this report.